

**Bilel Krichen: Spectral Theory for Linear Operators: Demicompactness
and Perturbation Theory, CRC Press, 2025.**
**ISBN: 9781032936352, EUR 210.00 (hardback) or EUR 189.00 (eBook),
320+xxiii pp.**

REVIEWED BY EDWARD L. BACH

This book attempts to present the spectral theory of demicompact operators. Demicompact operators were introduced by Petryshyn [4], who was interested in constructing fixed points for operators that are not necessarily norm-one contractions. In the last ten years, a number of papers have been published investigating spectral and perturbation theory for demicompact operators and related classes. Several of these papers are referenced in the book's extensive bibliography.

Some examples of demicompact operators are given in Chapter 5 of the book:

- (1) Every (not necessarily bounded) compact linear operator T with domain and range in a Banach space X ;
- (2) Every (not necessarily bounded) strongly continuous linear operator T with domain and range in a Banach space X ;
- (3) Every (not necessarily bounded) linear operator T with domain and range in a Banach space X , for which 1 is in the resolvent of T .

The book begins with an outline of the basic tools and results of functional analysis needed to develop and prove results in the spectral theory of demicompact operators (Chapters 1-4). Chapter 5 contains the definition of demicompact operators and related classes (weakly and relatively weakly Riesz and demicompact operators). Chapter 6 is a self-contained chapter dealing with nonlinear operators. The remaining chapters investigate aspects of the Fredholm and spectral theory of demicompact operators and related classes.

A researcher who is interested in the spectral theory of linear Banach-space operators or elements in Banach algebras will find a lot of familiar results and considerations here. Because the simple topological characterisation of demicompact operators apparently does not lead to a simple spectral characterisation (the examples above illustrate why there might be none), and because there is no operator ideal playing the role that the socle plays in Riesz and Fredholm theory (see [1]) for demicompact operators (as shown in, for example, [2, Remark 2.3], the demicompact operators are not themselves an ideal), it is likely there will be difficulties in developing the corresponding theory for demicompact elements of a Banach algebra. The present book does not attempt to develop such a theory, although [2] is clearly proceeding along these lines for relatively demicompact elements [2, Definition 2.7] of a Banach algebra.

This book could be used by researchers in functional analysis or operator theory who wish to familiarise themselves with the theory of demicompact operators and related classes. Alternatively it could be used, together with [1] and [3], in a class for postgraduates with a title such as “Spectral and Perturbation Theory in Banach Algebras”.

REFERENCES

- [1] B. A. Barnes, G. J. Murphy, M. R. F. Smyth and T. T. West: *Riesz and Fredholm theory in Banach algebras*, Research Notes in Mathematics 67, Pitman, 1982.
- [2] S. Chelly: *Stability of relative essential spectra involving relative demicompactness concept in Banach subalgebra*, Filomat, Vol. 37, No. 3 (2023), 891-903.
- [3] T. Kato: *Perturbation theory for linear operators*, Classics in Mathematics. Springer, Berlin. Reprint of the 1980 edition (1995).
- [4] W. V. Petryshyn: *Construction of fixed points of demicompact mappings in Hilbert space*, J. Math. Anal. Appl. 14 (1966), 276-284

Edward L. Bach studied Functional Analysis and Linear Operator Theory under Professor Trevor West at TCD, receiving a Ph.D in 1988. He has worked in Ireland as a software engineer, with a special interest in software control of safety-critical embedded systems, from 1989-2022 for Computer Applied Techniques in Malahide and from 2023-2025 with O.C.E. Technology in Belfield.

O.C.E. TECHNOLOGY LTD, NOVAUCD, BELFIELD INNOVATION PARK, DUBLIN, D04V2P1 IRELAND
E-mail address: `Edward.L.Bach@gmail.com`