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Marriage and Quaternions : Hamilton's Eureka Moment

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Abstract. This is a discussion of Hamilton's discovery of the quaternions, placed in the context of that joyous day. When during the twentieth century very negative and caricatural views on Hamilton and on his marriage became prevalent, his elated reports about his eureka moment lost their credibility. To show that the quaternions were born into a good and happy marriage, this article will start with a discussion of Hamilton's marriage as found in his two main biographies, published by Robert Graves in the 1880s, and by Thomas Hankins in 1980, from which it appears that Hankins was strongly influenced by the gossip which evolved after Graves' biography. Next, Hamilton's account of the discovery will be given, in which also Lady Hamilton plays her role. It will be followed by a chronology which shows that Hamilton's reports about the discovery are coherent and consistent. Then Hankins' reconstruction of the discovery will be discussed; he doubted some of Hamilton's claims, and in his version, there is no eureka moment. Yet, Hankins' version appeared to be based on an erroneous assumption of chronology, and therefore is less coherent than Hamilton's reports. This article will conclude with a lacuna in both Graves' and Hankins' biographies, namely that, next to his intellect, Hamilton also used his astounding memory for his private life. And with a plea for a new biography of Hamilton.

1. Introduction

It is common knowledge that, when in 1843 Sir William Rowan Hamilton (1805-1865) found the quaternions, Lady Hamilton (1804-1869) was walking with him along the Royal Canal near Dublin. Some authors of biographical sketches of Hamilton ignored this fact, due to the gossip about Hamilton that evolved in the twentieth century [11] perhaps believing the marriage was unhappy, or just unimportant.[1] Others felt sorry for her, having to wait and watch her husband carve some formula on Broome Bridge.[2]

This article will relate that exceptional event at the Royal Canal, and some hitherto little known details will be added. Hamilton's reports about how this discovery came to be appeared to be so consistent that it was possible to make a chronology of that now famous walk, a chronology that permits a determination of the near-exact moment of the discovery in which, in a flash of insight, triplets were replaced by quaternions. It will be asserted that the discovery happened within the context of a happy marriage, and that Lady Hamilton's presence that afternoon was a reason for delight.

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[1] See for instance J. Kuzmanovich, *William Rowan Hamilton* [users.wfu.edu], where Lady Hamilton is not mentioned. In a recent addition to the gossip, she "could not stand the strain of living with [Hamilton] and returned to live with her parents." J. S. Avery, *Lives in Mathematics*, 2021 [transcend.org].

[2] See for instance J. Baez, blog entry 2010, *Re: State-Observable Duality (Part 1)* [golem.ph.utexas.edu], "I pity his wife for having to stand by and watch him have his self-absorbed 'moment of genius'." Here, perhaps even more curious, it is mentioned that Lady Hamilton "eventually died."

To set such a description in context, it must be preceded by a discussion of Hamilton's two main biographies, the first one written by Robert Perceval Graves and published in the 1880s, the second one by Thomas Leroy Hankins, published in 1980. Graves' problems with the marriage of his adored friend will be discussed, and it will be shown that, strongly influenced by the contemporary gossip, Hankins had a less positive view on Hamilton than Graves had. Hankins openly doubted Hamilton's reports about how he had discovered the quaternions, yet it will be argued that Hankins' reconstruction of the discovery, in which he even changed details, was based on an erroneous interpretation of the chronology of that day. This article will conclude with aspects of Hamilton's ways of thinking that were missed by both biographers, but which throw a very different light on Hamilton and his marriage.

1.1. **The biographies and their consequences.** Graves' enormous biography consists of more than 2000 pages, filled with letters by Hamilton and his correspondents, to which Graves added comments. He mentioned that the Hamiltons had been "attached" spouses, and he called Lady Hamilton a "good woman" [2, 335]. Yet he was very critical of her; he believed that she should have made Hamilton stop drinking alcohol [2, 505], but had been too "weak of body and mind" to do so [10, 218-219, 381]. His criticisms and laments led to a train of gossip, in which during the twentieth century Hamilton became ever more alcoholic and even depressed, and instead of having been too weak to regulate her husband's drinking habits, Lady Hamilton became incapable of handling the entire household.

Hankins' biography is different; consisting of about 500 pages, Hankins gave his views on Hamilton and his work. He barely distinguished between opinions, conclusions and facts and, since in 1980 it was not yet possible to easily check statements online, his views came to be seen as the truth. This biography was especially detrimental for Lady Hamilton; Hankins began his section about her by stating, "Helen Bayly remains a shadowy figure in Hamilton's life in spite of all the references to her in the manuscripts" [9, 114].[3] Following the contemporary gossip [11, 16], he assumed that "when ill, or when Hamilton was away, when the burden of managing the observatory household became too great for her, she often went to stay with her mother at Nenagh or to [her sisters at] Dunsinea or Scripplestown" [9, 116],[4] something Graves never claimed. For three longer periods of time she indeed was away from home, but it had nothing to do with the household, and after 1842 she never again was away for longer than a few days.[5] Hankins closed the section by turning Graves' criticisms into a fact, "If [Hamilton] felt her weakness of health and character as a burden, it was a burden that he willingly accepted" [9, 126].

Thereafter the gossip about Lady Hamilton became extreme; nowadays she is described as a "pathologically shy and timid", "semi-invalid", "mentally limited" "local lass", a "country preacher's daughter" with "neurotic and probably slovenly tendencies" who was "unable to provide an orderly domestic regime," causing her husband to live in "squalor."[6] Such scorn further aggravated the views on Hamilton's private life, and consequently on his reports about the discovery of the quaternions; the fact that his wife had been present lost its lustre. This article aims to show that it was the trust

[3]There is some unavoidable overlap with the 2018 articles *A most gossiped about genius*, [11], and *Helen Bayly and Catherine Disney*, [doi.org/10.33232].

[4]Communal sleeping still was customary then [2, 273], and with only candles as sources of light, it is quite understandable that when Hamilton was away, Lady Hamilton preferred sleeping with her relatives in the neighbouring houses over sleeping alone in a necessarily very dark observatory.

[5]For why Lady Hamilton was away from home three times, and that it had nothing to do with the household, see [10, 178-188, 204-211].

[6]For these quotes, see the blog entry of 25 April 2020 [annevanweerden.nl].

and commitment between husband and wife that allowed for the peaceful atmosphere during their walk along the canal, in which Hamilton had his eureka moment.

1.2. **A psychological discovery.** One of the reasons for the stark difference between the positive view on Hamilton's marriage that emerges from his letters, and the current very negative view on it, is that both Graves and Hankins missed the fact that Hamilton, in 1832, discovered how to cope with feelings of loss [10, 119-120].

In August 1824 Hamilton, having just turned 19, had fallen in love with Catherine Disney and, in February 1825, he wrote a Valentine poem for her [1, 174]. Yet apparently becoming aware of Catherine's feelings for Hamilton, her family decided to advance the presumably already agreed marriage with her brother-in-law, William Barlow [12, 38]. Hamilton did not know that Catherine was coerced into this marriage, and assumed she married gladly; in a 'Farewell' poem, which he did not send to her, he prayed that her lot on earth would be a "younger heaven" [1, 185], as he had also written in the Valentine poem [1, 176]. Hamilton was devastated, and tried for years to bear his melancholy with calm, which at times was very difficult.

In December 1831 Hamilton fell in love with Ellen de Vere. After her indirect but certain rejection, he became even more melancholic than he had been before [1, 515], even to the extent that friends and family became worried [1, 528]. On 15 August 1832 he wrote to his friend Lady Campbell, who knew about Catherine Disney [12, 51],[7] "I had another affliction of the same kind and indeed of the same degree, except that my mind had been a little better disciplined to receive it [...]. I find that I am even now indulging too much my habit of dwelling upon painful recollections, instead of exerting myself to the utmost to control them by other thoughts" [1, 601], [10, 117].

Indeed, shortly before he wrote this letter, Hamilton had started to realise that he was living a "passion-wasted life" [1, 595]. He discovered how to cope with such feelings; that "action" instead of "meditation" was the "divine specific against Despondence" [10, 271],[8] which is what any therapist today would recommend to someone like Hamilton, a self-assured man who had an attached and secure childhood but did not know how to overcome such a loss. But that kind of talk therapy did not exist yet, and it indicates that Hamilton also used his enormous intellect for matters of life outside mathematics. In October 1832, his health having been greatly restored by this "power of hope" [2, 3], he found conical refraction, for which he would be knighted in 1835,[9] and fell in love with Helen Bayly [10, 121-122]. Although thereafter Hamilton could be troubled, worried, distressed, and at times melancholic, he never again drowned in such feelings.

That was shown again in October and November 1853, when Catherine Disney was finally able to tell him that she had wanted to marry him but had been coerced to marry Barlow, and died some weeks later [12, 83]. Hamilton became utterly distressed, now knowing that he had lost her to an unhappy and even forced marriage, yet this time he did not become melancholic; he started to write very many letters, according to Hankins to "unburden himself of grief" [9, 353]. Hankins concluded that it had "salutary effects" [9, 357], and apparently around June 1854 Hamilton was his happy self again [10, 307]; in those socially very strict years not having been able to talk about it, writing all these letters had been his "action instead of meditation" against despondence.

[7] In those times, reputations were easily destroyed. Only some people very close to Hamilton knew her name until in 1885 Graves revealed it to the world [2, 708], [3, 662].

[8] Hamilton wrote this to Catherine Disney in 1848 as a consolation [2, 611]. It did not help; at that time he still did not know that she had been forced to marry, but could not reveal that [12, 69]. In February 1833 Hamilton also gave this advice to his then wife-to-be, "the will has an influence [against gloomy fits]: there is such a thing as indulging in dejection, and on the other hand a persevering resolution against such indulgence effects much in time" [2, 22]. In Helen Bayly's case, this may have been very helpful advice; no biographer doubted the happiness of their early years.

[9] 'Mrs. Hamilton' then became 'Lady Hamilton', as she will be called hereafter.

But his friend Aubrey Thomas de Vere, Ellen de Vere's younger brother whom Hamilton had "won from the wreck" of her rejection [1, 511], as well as Graves and Hankins, missed this life-changing discovery. De Vere philosophised about passion and suffering [1, 615]; he loved the poems following Hamilton's loss of his sister, and about his love for Helen Bayly and the fear she also would reject him. In 1879 he wrote to Graves, "I suppose if [Hamilton] had not married [Helen Bayly in 1833] that remarkable outburst of poetry which characterized the year 1832 would have continued, and left large and important poetical results behind it."[10] That Graves missed it is surprising, because he knew that Hamilton more often had been occupied with "analysing his own character" [2, 247], [10, 189-190]. Yet he sometimes openly questioned Hamilton's views on himself [10, 190-192], and at times even disagreed with the choices Hamilton made for himself [10, 115], though always within the context of his adoration for his friend. That Hankins missed it appears from his conclusion that Hamilton "was saved from despair by always being able to work" [9, 112, 116].

2. The Hamilton marriage

William Rowan Hamilton and Helena Maria Bayly had met in the townland of Scribblestown, about eight kilometres northwest of Dublin city centre, where Hamilton lived at Dunsink Observatory since October 1827. Helen Bayly was from Nenagh, Co. Tipperary, and apparently regularly visited two of her elder sisters, Jane and Penelope Bayly, who also lived at Scribblestown [2, 2]. The sisters had married two brothers, Henry and William Rathborne; Jane and Henry lived at Dunsinea, Penelope and William at Scripplestown House. Both houses were only about a kilometre away from the observatory, and Hamilton loved to walk in their fields [1, 443], [2, 183].[11]

The Bayly family was from Ballinaclough, Debsborough, a few kilometres southeast of Nenagh,[12] and it is said about them that "unlike the national stereotype of the time of landlords being part of an elitist landed gentry, the Bayly's were known in Ballinaclough as good and fair landlords. They were considered to be the poor man's friend."[13] Lady Hamilton's father, Henry O'Neale Bayly, was an alumnus of Trinity College Dublin,[14] and, having become a clergyman, had lived since 1803 with his wife Anne Penelope née Grueber in Nenagh, where he was rector[15] and where Helen Bayly was born in 1804. Apparently around 1820 they built Bayly Farm, close to Debsborough House, where Henry's brother William Bayly lived.

Henry and Anne had twenty-three children, but not all names are known; some of them may have died young.[16] Henry died in 1826 at Scripplestown, the home of his daughter Penelope Rathborne, Anne died in 1837 at Dunsink Observatory, the home of her daughter Helen Hamilton.[17] From Graves' biography it seems that Hamilton had befriended especially William and Penelope Rathborne; although the Hamiltons also

[10]This letter is not online. It is kept at the Hesburgh Libraries of the university of Notre Dame. See the blog entry of 31 March 2018 [annevanweerden.nl].

[11]For a map of Scribblestown, with the observatory and the neighbouring houses, see [10, 158].

[12]For the townland Ballinaclough or Ballynaclogh, see Placenames Database of Ireland [logainm.ie]. For the Bayly Estate, including Bayly Farm and Debsborough, see Landed Estates [landedestates.ie]. For *Lady Helena Maria Hamilton's Bayly descent*, see [annevanweerden.nl].

[13]Silvermines Historical Society [silvermineshistoricalsociety.com].

[14]*Admissions Records* Trinity College Dublin p. 24 [doi.org/10.48495], also *Alumni Dublinenses* p. 51 [doi.org/10.48495].

[15]In the Anglican church, then the Established Church, rectors, vicars and curates were parish priests, with rectors the highest in status. Which priests were ordained depended on the parish.

[16]See William Edwin Hamilton's *Peeps at my Life* [archive.org]. For Helen Bayly's known siblings, see A. Young's 2021 blog entry *N is for Nenagh* [anneyoungau.wordpress.com].

[17]The British Newspaper Archive, 4-7 February 1826 [britishnewspaperarchive.co.uk], and 5-6 October 1837 [britishnewspaperarchive.co.uk].

were in close contact with the Rathbornes of Dunsinea [2, 273], [3, 102], Hamilton called William Rathborne his "private friend" [2, 101]. In 1831, Hamilton sent an invitation to Penelope Rathborne for tea [1, 442], asking her to bring her mother and sister with her; this is the first time Helen Bayly is mentioned in Graves' biography.

2.1. **Requisites.** Starting his section about Helen Bayly [9, 114],[18] Hankins gave parts of letters written by Graves in 1873 to Ellen de Vere, from which it appeared that Graves had come to loath Lady Hamilton; he wrote about her "poverty of mind" [9, 114, 414]. That may have started when, after Hamilton's death, Graves began to read the family papers to prepare the biography; he had been far from negative about the marriage in his 1842 *Portrait* of Hamilton,[19] and even as late as 1858, when he was visiting the observatory, he had "drank the health of Lady Hamilton" [3, 548].[20] Now knowing she had only written a few, rather short letters, the highly literary Graves must have been dismayed. From her letters it was very clear that she did not like writing them, she just wrote them because Hamilton wanted to receive letters from her [9, 119].

For Hamilton that was enough. Literacy was not what he had been looking for in a wife; his highest values were piety and truthfulness, and his requirement was that she was interested in his work [10, 152, 182].[21] He fell in love with Ellen de Vere because of her enthusiasm about astronomy [1, 459, 460], and he had stopped contemplating marriage with Louisa Disney, a younger sister of Catherine, because of her disinterest in science [9, 56]. He must have been well aware that he could be so deep into his "mathematical trances", as he called them when writing to Aubrey de Vere [1, 517], or in the "pains and pleasures of thought-birth," as he wrote in an ante-nuptial letter to Helen Bayly [2, 22], that she would often be unable to reach him even if he was close-by, or would have to go to bed alone; then a quite uncommon expectation of marriage.

Helen Bayly was everything he asked for: pious, truthful, and interested in his work [2, 14, 121], moreover, she had people she trusted in the neighbourhood. Yet having visited her sisters regularly, she must have seen how Hamilton lived his daily life at the observatory. Next to periods in which he was immersed in his mathematics and worked through the night, he was regularly visited by eminent people [1, 573], who usually brought their own staff [10, 175], which made managing that household a difficult task. Hamilton's three sisters Grace, Eliza and Sydney still took care of it [1, 603],[22] but when she would marry him, his sisters would leave the observatory, as was common then, and Helen must have doubted whether she would be able to cope with that.

She could very suddenly fall very ill [10, 197-198], and the medical treatments of those days such as calomel, raising blisters and shower baths[23] hardly alleviated her situation. Because visits often lasted for days, and sometimes even weeks, having to undergo such treatments while having eminent guests, or being a guest at someone else's

[18] The woman in the photo indicated as Helen Bayly [9, 123], was Hamilton's daughter and youngest child Helen Eliza Amelia.

[19] *Our Portrait Gallery – Sir William R. Hamilton*, The Dublin University Magazine, 19 (1842), 94-110 [babel.hathitrust.org].

[20] For placing Hamilton's remark on that page in context, see [10, 475].

[21] That Helen Bayly was truthful can for instance be seen in the ante-nuptial letters, when she refused to become "patient Griselda," which Hamilton called "courageous and candid" [10, 150]. Moreover, in a sonnet written in November 1832, Hamilton called Catherine Disney, Ellen de Vere and Helen Bayly "Sweet Piety, Enthusiasm, Truth," respectively [2, 4].

[22] The fourth and youngest sister was Archianna. Their mother Sarah Hamilton née Hutton died in May 1817 [1, 14], their father Archibald in December 1819. From 1818, Archianna lived with maternal relatives in Ballinderry [1, 65, 190].

[23] Shower baths had been installed at the observatory [9, 415]; bathing was recommended for "preserving health," and for "the cure of diseases." See p. 1211 of *An Encyclopædia of Domestic Economy* [archive.org]. For the shower baths, see p. 1220.

house, may have been a terrifying prospect. She also could not guarantee to uphold a high society Victorian life, full of dinners, parties and balls, and she asked for a "retired life at the Observatory" [10, 141, 387]; when he accepted that, she decided to marry him. She thus gave him a comparatively quiet life at the observatory, and created the circumstances in which he could do his mathematics the way he had to.

2.2. **"Layers of love and Algebra".** William Hamilton and Helen Bayly married on 9 April 1833 in Ballinaclough Church, less than two kilometres south of Bayly Farm.[24] Hamilton was very happy; a week before the marriage he had written from Bayly Farm to his former pupil Lord Adare, "I fear I may have forgotten something amid my alternate layers of love and Algebra, as I have lately been busy in both" [2, 42]. About two weeks after the wedding they went to Dunsink Observatory [2, 43], where the new Mrs. Hamilton had a difficult start. She fell ill in the summer and, not yet knowing she was pregnant since August [10, 170], in September she became "alarmingly ill" [2, 60], while all that time the observatory was being renovated [10, 162].

But eventually she felt at home at Dunsink; in January 1834 uncle James Hamilton wrote from Trim, "Your last letter was very gratifying, excepting the account you give of the health of your amiable wife. ... I know really scarcely anyone who seems more universally to have "won golden opinions" from her extensive circle of acquaintance, as letters from, and casual communication with, almost every friend we have testify. Of her present guests and your aunt she has won hearts – and hearts not easily won. I am preparing to yield mine, or rather have done so" [2, 72].[25]

3. An "obscuration" according to Graves

In the aforementioned 1879 letter to Graves, De Vere had also remarked, referring to a sonnet written by Hamilton on 16 June 1837 [2, 199], that "up to that time, A.D. 1837, Hamilton's affection for his wife had not waned. Indeed I do not know that it ever did,"[26] and that was something Graves could not ignore in his biography. Having introduced her at the beginning of the second volume as having been of "pleasing ladylike appearance" [2, 2], when describing 1840, Graves wrote that "[Hamilton] remained to the end of his life an attached husband, as Lady Hamilton remained an attached wife, as well as a good woman" [2, 335]. Yet he wrapped every positive remark about her in warnings and indications of doom, of which the most strange is his perceived "obscuration" of Hamilton, leading to a twisted and far-fetched lament [2, 334-335], [11, 5].

In November 1839 Lady Hamilton became pregnant again, and around the same time she started to suffer from what was called a "nervous illness".[27] From spring to August 1840 she lived in lodgings in Dublin, where Hamilton frequently visited her [10, 205]. After the birth of their daughter, who was born at home in August, she lived at Scripplestown, presumably to breastfeed her baby, and from February 1841 until January 1842 with a sister in England [2, 333], [10, 205-209]. Graves believed that from 1840, while it "gradually spread over some years, and for considerable time scarcely to be taken note of," Hamilton's personality changed, and that he became a "solitary worker" [2, 335]. Stating that before this "obscuration", in Hamilton's home "the strictest temperance reigned" [2, 505], Graves saw a "relaxation of domestic order" as the cause of the obscuration, the relaxation having been "a want of governance" by Lady

[24]Ballinaclough Church [historicgraves.com].

[25]After the deaths of his parents, "Uncle James" from Trim and his cousin, "Cousin Arthur" Hamilton from Dublin, became father figures for Hamilton.

[26]See the blog entry of 3 May 2020 [annevanweerden.nl].

[27]For Lady Hamilton's so-called "nervous illness," and what Hamilton's ante-nuptial promises about a retired life at the observatory had to do with it, see [10, 384-398].

Hamilton,[28] which led to the "long unfelt and unrecognised danger" of the "insidious habit" of drinking porter, in order to "dissipate chill" when at night "he desisted from the work of observing" [2, 334-335]. Graves had "learned on good authority" that such a habit caused "cravings" [2, 506], and he therefore believed that it led to a "humiliating" event in 1846, where Hamilton seemingly drank too much [2, 335, 507].

Graves' assumption that around 1840 Hamilton still regularly observed at night is strange because, from the first volume of his biography, it is obvious that after his early years at Dunsink Hamilton had to stop observing regularly because it "rather seriously affected his health" [1, 285, 358, 409]. Moreover, he must have known that Hamilton did not drink porter to "dissipate chill" because in 1866, the year after Hamilton's death, his eldest son, William Edwin, sent memoranda to Graves, and from one of them it appeared that, at some unknown time, Hamilton started to "take porter in small sips" when after many hours of concentrated mathematical work he became tired, but wanted to finish "a task in which good progress had been made" [3, 239], [10, 428-429].[29] It had nothing to do with observing in the cold; this was the highly concentrated mathematical work Lady Hamilton knew about before she agreed to marry him. She obviously granted her husband his "fascinating investigations" [1, 364], [2, 480], including his sipping porter. Graves furthermore combined two completely different periods, the first lasting from late 1839 until January 1842, the second from October 1843 until February 1846, into one continued process of "obscuration".

The first period started with Lady Hamilton's illness; Hamilton was very worried, and could hardly work. Moreover, in December 1840 his beloved Cousin Arthur Hamilton died [2, 331]. Early in 1841 Graves, who lived in England, visited Hamilton at the observatory, because he was writing his aforementioned *Portrait* of Hamilton, and he must have seen that Hamilton was not doing well. In May 1841 Hamilton wrote a depressed letter to his former tutor Charles Boyton, in which he mentioned to have been "driven from mathematics" [2, 332-334], and in August 1841, in a letter to Graves, Hamilton mentioned "many anxieties, more trying perhaps than the imaginative afflictions of early life" [2, 346].[30] This letter must have added to Graves' worries about Hamilton, which would explain why he believed that Hamilton's personality was changing [2, 335].

Graves wrote that when, in January 1842, Lady Hamilton had recovered so well that she could come home again [10, 211], Hamilton "immediately, with renewed cheerfulness, resumed his mathematical studies" [2, 361]. It is very likely that they had talked about the causes of her illness [10, 390], and even though every now and then Hamilton mourned the loss of Cousin Arthur, life was good, and the marriage happy again. Hamilton's sister Sydney, who had run the household during Lady Hamilton's absence, stayed until spring 1843 [2, 334, 414], more than a year after Lady Hamilton's return.

The second period started when, in October 1843, Hamilton discovered the quaternions. Combining that time frame with Graves' statements that after 1840 Hamilton became a solitary worker, but that the "obscuration" took some years and was hardly noted at first, it becomes very plausible to suggest that it actually started with the quaternions. After his discovery, the frequency of Hamilton's mathematical trances greatly increased, and he did become a solitary worker because no one could keep up with him. For a time he was nearly unstoppable; in November 1844 even his friend John Graves, a former college classmate and eldest brother of Robert and Charles Graves,[31] became worried that he would become overworked [2, 463], [10, 403].

[28] It must be noted that the "relaxation" was not about managing the household staff, that had been a temporary problem, when Lady Hamilton still was "expected soon to return" [2, 334].

[29] Porter is a dark malt flavoured beer, full of sugars. It is easy to imagine that when he became tired, Hamilton felt better because of the sugars in the porter.

[30] Here Hamilton doubtless referred to having lost Catherine Disney and Ellen de Vere.

[31] John Thomas Graves lived in England, and was a jurist and a mathematician.

Not having been a mathematician, Graves either did not recognise the importance of the quaternions, or he may have thought he needed this lament in order to prepare his readers for the "humiliating" event which happened in February 1846 at a meeting of the Geological Society, in which Hamilton seemed to have drunk too much, and which started the Dublin gossip about him [10, 414-417]. After the event, following advice from Graves' brother Charles, Hamilton abstained from alcohol for two years, but between 1848 and most likely 1851 [10, 424] he again drank in public, when enjoying social gatherings after long abstemious periods of hard work, and therefore perhaps a little less accustomed to the effects of alcohol than he had been before. It led to a further intensification of the Dublin gossip, and another lament by Graves [2, 632].

Around 1851, Charles Graves warned Hamilton about his reputation for a second time, and although this time he did not abstain, from then on Hamilton only drank very moderately [10, 422-428]. Even Graves was not entirely unsatisfied, writing that Charles Graves' "strengthening counsel was not without good effect" [2, 632], yet he again wrapped this positivity in criticisms. Combined with omitting to state explicitly that although Hamilton sometimes drank much[32] he was never drunk [9, 224],[33] he not only destroyed Lady Hamilton's reputation, but with it that of his admired friend.

To put Graves' lamentations into perspective, it must be realised that Graves never saw Hamilton skip meals or work through the night. He hardly knew what Hamilton's daily life looked like; from 1833 to 1864, therefore during nearly the entire duration of Hamilton's marriage, Graves lived in England, and when he did visit the observatory it was as an eminent guest. Moreover, he found himself in a very difficult position, having to publish, in the 1880s, during the times of temperance,[34] a biography about a friend who had loved the "pleasures of the table" [2, 527], [10, 418]. It was not an option to leave the 1846 event out; because of the Dublin gossip everyone knew about it, and he would lose his credibility. But at the same time, as an active clergyman, he may not have seen any possibility of defending his friend while simultaneously supporting his brother Charles Graves in his attempts to diminish the use of alcohol in Ireland.[35]

Graves therefore only once defended Hamilton, when he wrote about the period of Hamilton's resumption of drinking between 1848 and circa 1851, "Suffice it now to say that a most exaggerated notion of his weakness, of the degree to which he yielded to it, and of the number of his lapses, became prevalent" [2, 506]. Yet Graves did something remarkable when he wrote his pages about what had happened at the Geological Society in 1846; he gave a part of the account which Hamilton had sent him, that he had taken "what he was told was only a moderate quantity of wine" [2, 506]. Graves thus allowed his readers to conclude that Hamilton had not been drunk without literally saying so.

On six or seven pages, scattered through the biography, Graves lamented about his friend's alleged "cravings", and how it was all Lady Hamilton's fault. He could not know that readers outside Dublin, and living long after everyone featuring in the biography had died, would only remember his laments, that his other 2000+ pages, about what a wonderful man Hamilton had been, were soon forgotten. That was the unavoidable consequence of Graves' choice to write a biography which was neither about

[32]At the 1842 meeting in Manchester of the British Association, Hamilton met Carl Jacobi [2, 386-387]. They were "left with a deep mutual respect for each other's drinking prowess." See p. 3 of *Sir William Rowan Hamilton (1805-1865)* by J. H. Reid, 1965 [articles.adsabs.harvard.edu]. Very unusually for 1965, Reid called the Hamilton marriage "very happy."

[33]For a discussion about the 1846 event at the Geological Society, and that Hamilton had not been drunk, see *A biographer's opinion as primary source* [youtube.com].

[34]See *Temperance Movement in Ireland* [en.wikipedia.org]. Or, *The influence of the 1880s temperance struggles on Hamilton's posthumous reputation* [youtube.com].

[35]In the 1840s, Charles Graves was a fellow mathematician at Trinity College Dublin; in the 1880s, when the biography was published, he was Bishop of Limerick.

Hamilton's work, nor about his daily life. It was instead one long defence against the Dublin gossip [1, v], in which Graves tried to show how all eminent people he could think of [2, 656-657] admired his morally high-standing friend.

4. Hankins' views on Hamilton's private life

Following the publication of Graves' biography, in an ever worsening train of gossip, Lady Hamilton came to be described as a semi-invalid who could not manage the slovenly servants. It had caused Hamilton to become sad and lonely and, giving in to his cravings, he had started to drink at a dangerous level [11, 14-16]. Yet it still was all Lady Hamilton's fault until, in 1980, Hankins took it to a higher level. Graves had been unhappy with some important choices Hamilton made in his life; he should have married Ellen de Vere [10, 115], should have forswore alcohol, and should not have given himself over to his mathematical trances at the cost of the "healthful order and beauty [of] the course of his daily life" [1, 506]. Yet further aggravating Graves' view on Hamilton, Hankins was certain; Hamilton only loved Catherine Disney [9, 39, 347].

Hankins apparently did not realise that until 1848 Hamilton had not known how extremely unhappy Catherine Disney was [12, 70], that only shortly before her death in 1853 could she tell him that she had wanted to marry him, but that she had been forced by her family to marry Barlow. And not having recognised Hamilton's 1832 psychological discovery, that he had found that "action instead of meditation" was the remedy against despondence, Hankins took Hamilton's understandably very distressed writings after these two shocking revelations for an everlasting truth.

He moreover dismissed what Hamilton himself wrote about his feelings for Ellen de Vere and Helen Bayly. That, as mentioned above, what he felt for Ellen de Vere had been an "affliction of the same kind and indeed of the same degree" as that of Catherine had been. And that his ante-nuptial poems show how much in love he was with Helen Bayly [2, 3-7], how afraid he was that she would not accept him [2, 7-16], and how happy when she did; finally being certain she would marry him he wrote, "how full of silence is deep happiness" [2, 27], [10, 157]. How honest Hamilton was in his poems appears from the fact that, while holding to the conviction that poetry must come from true feelings [10, 80], he sent his ante-nuptial poems to Samuel Coleridge, in his accompanying letter calling them "love poems" [2, 37], [10, 146].

Hankins instead concluded that Hamilton had been a hopeless romantic [9, 347], and in one stroke added a new branch to the gossip; it was concluded from his biography that Hamilton's drinking alcohol had all been about Catherine Disney. Completely passing over Graves' remarks that the "obscuration" had "gradually spread over some years," and had been "long unfelt and unrecognised," in his section 'From Algebraic Couples to Quaternions', Hankins assumed that already in 1840 or 1841, when "seriously depressed" by his wife's absence and the death of Cousin Arthur, Hamilton started to "suffer from his first difficulties with alcoholism" [9, 287].[36]

In his section 'Catherine Disney Barlow', Hankins returned to the subject, now indicating 1844 instead of 1840 or 1841 as the year of the first problems with alcohol;

> [In 1844] Hamilton first began to have trouble with alcohol. The following year, in 1845, his old college friend Thomas Disney paid a visit to the observatory and brought Catherine with him. [...] He left little record of the 1845 visit, but it must have been upsetting in the extreme. In February [1846] his problems with alcohol became public knowledge when he collapsed and became violent at the meeting of the Geological Society [9, 348].

[36] Hankins did not literally give a year, but thereafter he quoted a sentence from a letter by Hamilton to John Graves, written in October 1840 [2, 328], and Cousin Arthur died in December 1840.

By putting the sentence about 1846 so directly after the one about 1845, Hankins implied a direct connection between Catherine's visit in 1845 and the event in 1846.[37] However, such a continuous path of drinking more and more, lasting from 1844 to 1846, or even from 1840 to 1846, is not likely at all.

From about 1839 [2, 284] until early 1846 [9, 238], Hamilton had been attracted by the "high church" Oxford Movement, which "argued for the reinstatement of some older Christian traditions of faith and their inclusion into Anglican liturgy and theology."[38] Graves had written that, in the summer of 1845, Hamilton had experienced "religious contrition" over having drunk more than he had judged right, even though according to Graves he "was perfectly clear in mind and in possession of all his powers" [2, 491], [10, 409]. Moreover, in his section 'In and Out of the Oxford Movement', Hankins wrote about 1845, referring to Graves' third volume [3, 236],

> This was the year that Hamilton's friends noted his excitability over religious matters. He became much more strict in all religious observances. [...] These were what Hamilton's son William Edwin later called his "high church days," when Hamilton felt strongly about proper religious observance. Even their greyhound "Smoke" was subject to special religious discipline and received a beating for chewing on an old Book of Common Prayer in the library, the implication being that chewing a sacred book deserved a more severe punishment than chewing a nonsacred one [9, 238-239].

The idea that an openly truthful man as Hamilton, adhering so strict to religious observances that he even punished his beloved dog, had been drinking too much from 1840 or 1844 until February 1846, is utterly illogical. Not in the least because during his "high church days" Hamilton "read prayers in the centre of his assembled household" [3, 236], which means that if in that particular period he had been drinking too much, perhaps even seeking to do so secretly, everyone in his household, or living on the Dunsink premises for that matter [2, 311], would have judged him to have been a hypocrite, but no trace of that was found, rather the contrary [10, 49 fn. 64, 502].

When describing the February 1846 event at the Geological Society, Graves had repeated Hamilton's own view on what had happened, allowing his readers to come to their own conclusions. But in Hankins' version, all these nuances were gone. Moreover, making some mistakes led to a far more drunken impression than Graves had given. Hankins assumed that during the dinner, Hamilton had "talked excitedly" about "a plan to use the instruments at the observatory to detect earth movements" [9, 223], but what had happened was that that morning Hamilton had received the confirmation of this "suggestion of a new connexion between astronomy and geology" [2, 506], which explains why he had been so excited. Hankins further wrote that Hamilton had "fainted and tumbled down the stairs" [9, 223], yet that is not in Graves' description [2, 506-507], to which Hankins referred [9, 429]. Nor was Graves "close to the events" as Hankins suggested; he lived in England, and Hamilton had sent him an account of what had happened. Hankins did not repeat the "moderate quantity of wine," instead he suggested that the fact that thereafter Hamilton stopped drinking alcohol "would indicate that alcohol had been the cause of his collapse" [9, 223]. Not even Graves suggested that; shortly after the event Hamilton followed the advice of Charles Graves to stop drinking alcohol, not for his health, but for his reputation [2, 507, 527], [10, 415]. Hamilton then abstained for two years.

[37] That Hankins made this connection may not seem completely obvious here, but he directly made that connection when he described Hamilton's 1848 visit to Parsonstown, see p. 57.

[38] For the Oxford Movement, see Wikipedia [en.wikipedia.org].

In July 1848, Catherine Disney contacted Hamilton. That led to an upsetting correspondence, in which Catherine told Hamilton how terribly unhappy she was [10, 269-275]. After the conclusion of this correspondence, Hamilton visited Parsonstown, where Lord Rosse had built his enormous "Leviathan" telescope. After a few days having calmed down again, Hamilton drank a glass of champagne with Airy, then Astronomer Royal of England [10, 284, 418-420]. Hamilton left Parsonstown on 12 September, and on 5 October 1848 Catherine wrote to Hamilton that she wanted to commit suicide. But in 1853 Hamilton mentioned that he had received her suicide note in Parsonstown, and comparing these dates, Hankins supposed that Catherine wrote her note on 5 September instead of 5 October, that "Hamilton must have confused the two months in his remembrances" [9, 449]. He then concluded that Hamilton's distress about the suicide note had led him to end his abstinence, that "the internal torture of Catherine's suicide attempt must have been the real culprit – if a culprit was needed" [9, 350].

There are serious problems with Hankins' scenario. From letters Hamilton then wrote, it can be inferred that he did not read the note before 11 September, and by then Airy had already left Parsonstown [10, 278-285]. Hamilton left on the 12th, but it is very likely that he later returned to Parsonstown; on 16 September Saturn's moon Hyperion was discovered, and from 21 September to 5 October the circumstances to see it with the "Leviathan" were ideal.[39] It is therefore perfectly possible that Hamilton then and there received Catherine's note, and his drinking champagne thus had nothing to do with distress about it; he most likely had judged he had abstained long enough.

Despite his assertions, Hankins did not believe that Hamilton became an alcoholic,

> Hamilton remained on the edge of alcoholism for the rest of his life, but his condition was never as bad as the Dublin gossips would have it. There were no more incidents like the one at the Geological Society, and there is no indication that he ever lost his capacity for sustained mathematical work. The gigantic volumes of the *Lectures on Quaternions* and the *Elements of Quaternions* could never have been written by a man in an alcoholic stupor [9, 224].

But just as happened to Graves' positivity, which was wrapped in criticisms, Hankins' defences were sandwiched between doubts about Hamilton's private life [9, 224, 378-379]. Hankins' confident sentences about "trouble with alcohol," and having "remained on the edge of alcoholism," were the ones which were best remembered.

4.1. **Views on the marriage.** After the publication of Hankins' biography, the view on Hamilton and his marriage became caricatural. For instance Hankins' suggestion that Hamilton did not speak of love in connection with Helen Bayly when in 1832 he wrote to De Vere about her [9, 113], led to views of Hamilton as a hopeless case, drowning in alcohol because he had not married Catherine Disney. Yet not having spoken of love is only literally true for the first letter to De Vere, written on 7 November 1832 [1, 622], because five days later Hamilton wrote a second one,

> very lately I have begun again to associate Hope with Love, in the case of a person whom I have long known and deeply respected for eminent truth of character. [...] I felt myself induced by our friendship, and bound by our agreement, to apprise you (as I lately did) as soon as I perceived the dawn of a new love. How the seed has grown into a tree, the sonnets that I have just composed will show you [2, 3, 4-6].

In his section about Helen Bayly, having first given Graves' harsh opinion about her, Hankins defended her by commenting that "Graves' judgment was probably not fair" because he "idolized Hamilton, and was all too ready to blame his hero's weaknesses on

[39] See the blog of 22 February 2020 [annevanweerden.nl].

the lack of "controlling influence" from his wife" [9, 114]. But then he went even further than Graves had done in his biography. Graves had written that in 1832 Hamilton's "tenderer and warmer feelings" for Helen Bayly came "at a time when he had felt obliged to suppress his former passion" [2, 2]; Hankins wrote her off as a serious factor in Hamilton's life by stating that she "knew that Hamilton married her "on the rebound," so to speak, from two previous romances," which she could not ever replace [9, 115].

Hankins drew this latter conclusion from what he called "a revealing sonnet" [9, 114-115], but in which, on the contrary, Hamilton showed Helen Bayly why he would not hide his former loves from her, "Praising a new love to dispraise the old, As if I had before but false tales told" [2, 4]; after all, his former "loves" were not relationships which had gone awry, he had loved and lost them. Hamilton was completely open about his former loves, and he doubtless was of the opinion that he had to be; Augustus De Morgan, who corresponded with Hamilton from 1841 until his death in 1865, remarked about Hamilton's extreme honesty, "In the matter of right and wrong, Hamilton was very simple-minded. To say he was truthful would be only a part of the truth; his aptitude to entertain misgivings [...] made him often think it right to express his opinions to avoid the possibility of being misunderstood" [3, 218]. Throughout Graves' biography it is obvious that Hamilton valued truth very highly, and his absolute clarity about loving Helen Bayly as he had loved Catherine Disney and Ellen de Vere was why she was certain of his love for her.

Their mutual trust can be felt in Hamilton's description of the walk along the Royal Canal, that his wife "talked with [him] now and then, yet an under-current of thought was going on in [his] mind" [2, 434]. It is not really known what a eureka moment is or where it comes from, but it seems to emerge more often when someone is relaxed and a little distracted after a period of intense thought as, allegedly, Archimedes was in his bath. In Hamilton's case, it is easy to imagine how, when his wife joined him on his walk, the "under-current" which led to his eureka moment would have been interrupted if this marriage had been as bad as the gossip has it. Yet it was their shared faith and spirituality [10, 197], and high value in truth, that defined this marriage and created the peaceful atmosphere which can be found in the poems Hamilton wrote for and about his "wedded and beloved one", his "gentle" wife [2, 50, 270], and in the report about his discovery of the quaternions as they were walking together along the Canal.

5. The discovery of the quaternions; Hamilton's account

Hamilton left eight known reports about his discovery[40] of the quaternions,[41] of which the shortest one consists of two pages of a pocket-book, Figure 5.3, written on the afternoon of 16 October 1843 [2, 436]. There is an entry in Hamilton's 'Note-book 24·5', or as Graves called it, 'A 1829' [2, 439], dated 16 October 1843, written after Hamilton had returned home, hereafter called *Evening entry* [4, 103-105]; a letter to John Graves written on 17 October 1843, the day after the discovery, hereafter called *Letter to John Graves* [5, 489-495]; a paper in the *Proceedings of the Royal Irish Academy* which was communicated on the 13th of November 1843, hereafter called *Proceedings* [6, 424-434];

[40] In the *Letter to John Graves*, Hamilton wrote, as a footnote to the word "discovered", "The reader is requested to pardon this expression for the reason mentioned in another note" [5, 490]. Such a note was not found, but Hamilton invariably used the words "discovered" and "discovery" for the quaternions.

[41] "Previously, in 1819 or 1820, Gauss had set down the multiplication table for quaternions, but did not publish it; nor did he develop the algebra" [4, xv]. The exact reference given in [4], "Sitz. Göttingen Nachrichten 1898, p. 8" was not found, but on pp. 130-131 of *Mathematische Annalen* 1899, Band 51, Felix Klein wrote, "[Gauss] gibt die expliciten Formeln für die Zusammensetzung zweier Scalen (also die Multiplication zweier Quaternionen), wobei er die symbolische Schreibweise $(abcd)\,.\,(\alpha\beta\gamma\delta) = (ABCD)$ benutzt und ausdrücklich bemerkt, daß es sich dabei um einen nicht commutativen Process handelt!" [gdz.sub.uni-goettingen.de]. See also footnote 47.

an entry in one of Hamilton's manuscript books, written 18 July 1848, hereafter called *Book C., 1848* [2, 437-439]; a letter to Peter Guthrie Tait, written 15 October 1848, hereafter called *Letter to Tait* [2, 435-436]; a part of the preface to Hamilton's *Lectures on Quaternions*, dated June 1853, hereafter called *Preface* [7, (43)-(47)]; and a letter which Hamilton wrote to his son Archibald Henry on 5 August 1865, about a month before his death, hereafter called *Letter to Archibald* [2, 434-435].

Hamilton's version of how he discovered the quaternions will be described within the context of the above discussed positive view on his marriage. It would indeed have been logical for Graves to conclude that the "obscuration" he perceived had begun in 1843, after Hamilton discovered the quaternions and became captivated by them. Yet Graves was not a mathematician, and searching for personal or emotional reasons for the change in Hamilton,[42] he believed, as mentioned before, see p. 52, that it began in 1840, and that it "gradually spread over some years, and [was] for considerable time scarcely to be taken note of." But that means that, regardless whether Graves was right or not, in 1843 it did not yet play a role, or it was, in the worst possible scenario, still "scarcely visible." This justifies placing the discovery in the context of a happy marriage.

5.1. **Reminiscing.** Since 1830 [1, 366, 419], Hamilton had searched, "at intervals" [2, 439], for triplets, an extension of the complex numbers, $a + ib$, by an added second imaginary term, $a + ib + jc$; it was expected that these triplets could be used for geometrical descriptions in three dimensions, similarly to imaginary numbers in two dimensions. Hamilton wanted to "discover the laws of [their] multiplication" [2, 434], and he started his last search in the second half of September 1843. In 1865, a month before his death, Hamilton wrote in the *Letter to Archibald*,

> If I may be allowed to speak of myself in connexion with the subject [of the multiplication of vectors], I might do so in a way which would bring you in, by referring to an ante-quaternionic time, when you were a mere child, but had caught from me the conception of a Vector, as represented by a Triplet. [...] Every morning in the early part of [October], on my coming down to breakfast, your (then) little brother William Edwin, and yourself, used to ask me, "Well, Papa, can you multiply triplets"? Whereto I was always obliged to reply, with a sad shake of the head: "No, I can only add and subtract them" [2, 434].

That changed when, on Monday 16 October 1843, Hamilton was walking along the Royal Canal on his way to preside over a council meeting of the Royal Irish Academy [2, 434-437], of which he had become president in 1837 [2, 211]. Council meetings were held "on the first and third Mondays of every month, from November to June, inclusive, at half past three, P.M. or when specially summoned; and the Academy at large [meets] on the second Monday of November and December, and on the second and fourth Mondays of January, February, April, May, and June, at Eight, P.M.," as was announced every year in the *Dublin Almanac*.[43]

The council meeting of 16 October 1843, held during the annual adjournment, thus was such a 'summoned meeting'; those meetings were held on the same weekdays and times as the regular council meetings. Having to open the meeting at half past three, see Figure 5.1, Hamilton had left home in the early afternoon. The route he used to take while walking to Dublin is well known;[44] first walking across the sloping fields to

[42]Here Graves slightly differed in opinion with his brother John Graves, who did understand the mathematics, [2, 462-463], [10, 404].

[43]See the *Dublin Almanac* for 1843, p. 167 [babel.hathitrust.org]. Time is in Dublin local time, UTC -25m21s [dublincity.ie].

[44]As the 'Hamilton Walk', it is walked yearly in commemoration of Hamilton's eureka moment [wikipedia.org]. For the route, see the map which was made for it [google.com/maps].

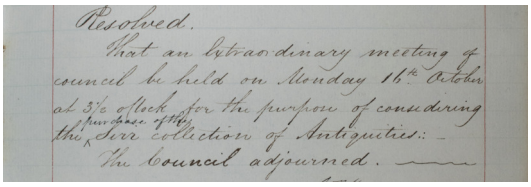
Resolved.
That an Extraordinary meeting of
council be held on Monday 16th October
at 3½ o'clock for the purpose of considering
the purchase of the Sirr collection of Antiquities.: –
The Council adjourned. ——

FIGURE 5.1. Minutes of the RIA council meeting of 2 October 1843. "Resolved. That an extraordinary meeting of council be held on Monday 16th October at 3 1/2 o'clock for the purpose of considering the purchase of the Sirr collection of Antiquities.: – The council adjourned." *By permission of the Royal Irish Academy © RIA*

the south, after having passed Dunsinea, where Lady Hamilton's sister Jane Rathborne lived, he turned left and walked along the Royal Canal. Hamilton must have been very deep in thought because he did not hear his wife coming, nor did he know where she came from, as appears from the *Letter to Archibald*, given below. This leads to the likely scenario, that when Hamilton passed by Dunsinea, Lady Hamilton saw him, and because she apparently also wanted to go to Dublin, she decided to catch up with him.

In the *Letter to Archibald*, Hamilton described their walk and his discovery,

> I was walking in to attend and preside, and your mother was walking with me, along the Royal Canal, to which she had perhaps driven; and although she talked with me now and then, yet an under-current of thought was going on in my mind, which gave at last a result, whereof it is not too much to say that I felt at once the importance. An electric circuit seemed to close; and a spark flashed forth, the herald (as I foresaw, immediately) of many long years to come of definitely directed thought and work, by myself if spared, and at all events on the part of others, if I should even be allowed to live long enough distinctly to communicate the discovery. Nor could I resist the impulse – unphilosophical as it may have been – to cut with a knife on a stone of Brougham Bridge, as we passed it, the fundamental formula with the symbols, i, j, k; namely,
>
> $$i^2 = j^2 = k^2 = ijk = -1,$$
>
> which contains the Solution of the Problem, but of course, as an inscription, has long since mouldered away [2, 434-435].

FIGURE 5.2. Sir William Rowan and Lady Helena Maria Hamilton depicted at Broome Bridge on 16 October 1843. A sand sculpture by Daniel Doyle, 2012, Dublin. The reverse side, in fact the front side, shows Hamilton "trying to understand dimensions" [steemit.com]. *Reproduced with permission.*

In the 1858 *Letter to Tait* Hamilton wrote,

> [The quaternions] started into life, or light, full grown, on the 16th of October, 1843, as I was walking with Lady Hamilton to Dublin, and came up to Brougham Bridge, which my boys have since called the Quaternion Bridge.[45] That is to say, I then and there felt the galvanic circuit of thought close; and the sparks which fell from it were the fundamental equations between i, j, k; exactly such as I have used them ever since. I pulled out on the spot a pocket-book, which still exists, and made an entry, on which, at the very moment, I felt that it might be worth my while to expend the labour of at least ten (or it might be fifteen) years to come. But then it is fair to say that this was because I felt a problem to have been at that moment solved – an intellectual want relieved – which had haunted me for at least fifteen years before [2, 435-436].

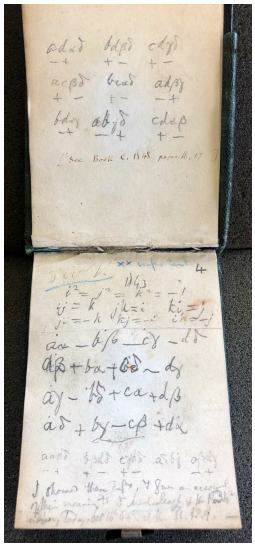

FIGURE 5.3. The pocket-book showing, at the top of the lower page, the "fundamental equations." "xx important" was written by Graves when he was preparing the biography, with his "blue pencil" as Hankins called it [9, xxi]. Beneath the terms on the upper page, Hamilton later inserted a note about his *Book C., 1848* [2, 437].
Courtesy of the Board of Trinity College, the University of Dublin.

And on 18 July 1848 Hamilton wrote in his *Book C., 1848* [2, 437],

> The pocket-book remains, given me by Helen in 1840, in which, while I was walking to town on the day thus referred to, I did actually pencil at the time, and just as I reached the Bridge here mentioned, the notes

$$\begin{gathered} i^2 = j^2 = k^2 = -1 \\ ij = k \qquad jk = i \qquad ki = j \\ ji = -k \qquad kj = -i \qquad ik = -j. \end{gathered} \tag{5.1}$$

5.2. **Quaternions.** In the *Letter to John Graves*, written on 17 October, Hamilton gave his course of thought the day before, and most clearly described his discovery. He had aimed for a system of triplets which would be "analogous" to "Mr. Warren's geometrical

[45] It has been suggested that calling the bridge "Brougham Bridge" was a family in-joke, see p. 618 of Charles Mollan, *It's Part of What We Are* [google.com/books].

representation of imaginary quantities,"[46] essentially the now common representation in a plane. With $i^2 = j^2 = -1$, the multiplication of two triplets, $(a+ib+jc)(x+iy+jz) = ax - by - cz + i(ay + bx) + j(az + cx) + ijbz + jicy$, raised the question what to do with ij. Hamilton considered $ij = 1$, or $ij = -1$, but then his prerequisite, that "the sum of the squares of the coefficients of 1, i, and j in the product [should be equal] to the product of the corresponding sums of squares in the factors" [5, 490], would not hold; his system would not obey this multiplicative property of lengths, which Hamilton called the "law of the moduli," and a geometrical representation as Mr. Warren's would not be possible.

Hamilton then calculated a square as the simplest case of a product, while suppressing both ij and ji, $(a+ib+jc)(a+ib+jc) = a^2 - b^2 - c^2 + 2iab + 2jac$. To obey the law of the moduli, it was necessary that the product of the sums of squares in the factors, here $(a^2 + b^2 + c^2)^2$, was equal to the sum of the squares of the coefficients of 1, i, and j in the product, which indeed is the case, $(a^2 - b^2 - c^2)^2 + (2ab)^2 + (2ac)^2 = (a^2 + b^2 + c^2)^2$. Therefore, in case of a square,

> the condition respecting the moduli is fulfilled, if we suppress the term involving ij altogether. [...] Behold me therefore tempted for a moment to fancy that $ij = 0$. But this seemed odd and uncomfortable, and I perceived that the same suppression of the term which was de trop might be attained by assuming what seemed to me less harsh, namely that $ji = -ij$. I made therefore $ij = k$, $ji = -k$, reserving to myself to inquire whether k was $= 0$ or not. For this purpose I next multiplied $a + ib + jc$, as a multiplier, into $x + ib + jc$ as a multiplicand, keeping still, as you see, the two factor lines in one common plane with the unit line [5, 490-491].

For such triplets, $ij = k$ and $ji = -k$ indeed cancels the k term; $(a+ib+jc)(x+ib+jc) = ax - b^2 - c^2 + i(ab + bc) + j(ac + cx) + k(bc - cb)$.

> [...] Confirmation of $ij = -ji$; but no information yet of the value of k. Try boldly then the general product of two triplets, and seek whether the law of the moduli is satisfied when we suppress the k altogether. Is $(a^2 + b^2 + c^2)(x^2 + y^2 + z^2) = (ax - by - cz)^2 + (ay + bx)^2 + (az + cx)^2$? No, the first member exceeds the second by $(bz - cy)^2$. But this is just the square of the coefficient of k, in the development of the product $(a + ib + jc)(x + iy + jz)$, if we grant that $ij = k$, $ji = -k$ [5, 492].

This is where, in the *Letter to John Graves*, Hamilton ended the pre-discovery calculations, yet in the *Evening entry*, which he wrote for himself, there are some extra steps.

Before discussing them, it must be remarked that there is a seemingly confusing part in the *Evening entry* as given in [4], namely, that Hamilton first wrote down that he had conceived "that the product ij might be equated to a new imaginary, say k," and then that he "still [...] thought it possible that ij might be equal to zero" [4, 104]. He then wrote, "I believe that I remember now the order of my thought" and corrected himself, starting anew with the two triplets having their factor lines in one common plane with the unit line; this is written out in the quote on p. 63. The confusion disappears when it is realised that Hamilton was writing this for himself, and therefore had no reason to strike errors through, nor could he tear out pages of his notebooks; he usually also wrote on the reverse side of the pages [9, 399].

[46] John Warren, 1828, *A Treatise on the Geometrical Representation of the Square Roots of Negative Quantities*, [archive.org]. Hamilton's geometrical contemplations have been omitted here, they can be found in, for instance, [5, 10-13] and [6, 424-434].

In the *Evening entry* already having concluded that $ij = -ji$, Hamilton remarked, "trying to remember this evening my course of thought in the morning," that he had "even thought it likely though odd" that $ij = 0$. In the *Letter to John Graves* Hamilton showed how $ij = 0$ came from squaring a triplet, but in the *Evening entry* he wrote,

> I believe that I remember now the order of my thought. The equation $ij = 0$ was recommended by the circumstance that $(ax - y^2 - z^2)^2 + (a + x)^2(y^2 + z^2) = (a^2 + y^2 + z^2)(x^2 + y^2 + z^2)$ [4, 104].

This indeed follows from $(a+iy+jz)(x+iy+jz)$, as Hamilton also wrote in the *Letter to John Graves*, the only difference is that there he used $(a + ib + jc)(x + ib + jc)$ to show that the k term, or the ij term, "still vanished," and he did not so succinctly order the terms.

Hamilton continued, just as in the *Letter to John Graves* using $ij = k$, $ji = -k$ for two general triplets and supposing the k term is cancelled again,

> I therefore tried whether it might not turn out to be true that
>
> $$(a^2 + b^2 + c^2)(x^2 + y^2 + z^2) = (ax - by - cz)^2 + (ay + bx)^2 + (az + cx)^2;$$
>
> but found that this equation required, in order to make it true, the addition of $(bz - cy)^2$ to the second member. This forced on me the non-neglect of ij; and suggested that it might be equal to k, a new imaginary. For a while I thought it likely that k^2 might be equal to 1; but this supposition $k^2 = 1$, combined with $i^2 = j^2 = -1$, would give
>
> $$(a + ib + jc + kd)(\alpha + i\beta + j\gamma + k\delta) = a\alpha - b\beta - c\gamma + d\delta + i(\&c.) + j(\&c.) + k(\&c.); \tag{5.2}$$
>
> in which the part multiplied by k must involve the terms $a\delta + d\alpha$, unless we choose to give up [...] certain very simple and fundamental principles of ordinary algebra [...]; but in the square of the modulus, (that is, in the sum of the 4 squares of the coefficients of 1, i, j, k) [which he later wrote out on the lower page of the pocket-book, Figure 5.3], this part of the coefficient of k would give the term $+2ad\alpha\delta$, while, if $k^2 = 1$, the coefficient of 1 would give the same term $2ad\alpha\delta$ repeated; thus there would be no mutual destruction of these two terms. But if we suppose $k^2 = -1$, the real part (so to speak) is $a\alpha - b\beta - c\gamma - d\delta$; and this gives $-2ad\alpha\delta$, destroying $+2ad\alpha\delta$, which latter arises from $a\delta + d\alpha$. Thus I was led to assume not only $i^2 = -1$ and $j^2 = -1$, but also $k^2 = -1$, and $ij = k$, $ji = -k$ [4, 104].

It thus appears that already having assumed $ij = k$, $ji = -k$, which he had also mentioned in the *Letter to John Graves*, Hamilton now had added $k^2 = -1$; it can therefore be concluded that finding that the square of the new imaginary k should also be equal to -1, led to the flash of insight.

5.3. **The discovery.** Hamilton described the moment of discovery in the *Letter to John Graves*, which he continued writing,

> And here there dawned on me the notion that we must admit, in some sense, a fourth dimension of space for the purpose of calculating with triplets; or transferring the paradox to algebra, must admit a third distinct imaginary symbol k, not to be confounded with either i or j, but equal to the product of the first as multiplier, and the second as multiplicand; and therefore was led to introduce quaternions such as $a + ib + jc + kd$, or (a, b, c, d) [5, 491-492].

Although in the *Letter to John Graves* the moment of discovery is very clear, in the *Evening entry* that is far less so. Yet comparing these two reports, both times the "new imaginary" follows from recognising the extra term $(bz - cy)^2$. It can therefore be suggested that, already feeling certain about $ij = k$, $ji = -k$, while still at home Hamilton had recognised the extra term which, as he wrote in the *Evening entry*, "forced on him the non-neglect of ij; and suggested that it might be equal to k, a new imaginary."

This seems the most likely stage in the calculations at which Hamilton had to lay down his pen and perhaps take the cat from his shoulder [3, 235]. He may have had luncheon, and started walking, meanwhile contemplating that the square of this k might be equal to 1 because $i^2 = j^2 = -1$ and therefore perhaps $ij = 1$ or $ij = -1$, as he wrote in the *Letter to John Graves* [5, 490]; at that time, k was still a product of i and j, and he was still searching for triplets. He therefore started to calculate the still unnamed quadruplet, equation (5.2), thus including the extra term which was connected to k. There was nothing really new in trying a quadruplet because already in 1841, Hamilton had written to De Morgan, "if my view of algebra be just, it must be possible, some way or other, to introduce not only triplets but polyplets" [3, 247].

It can be assumed that these calculations were the "under-current" of thought Hamilton mentioned in the *Letter to Archibald*, quoted on p. 60; after Lady Hamilton had joined him, she talked with him "now and then," leaving enough room for such calculations, to which he was completely used, and still nothing was new. But just before reaching Broome Bridge he realised that the $2ad\alpha\delta$ term would not be cancelled if $k^2 = 1$, but it would if $k^2 = -1$, and then such a quadruplet would obey the law of the moduli; k was a "third distinct" imaginary, on equal footing with i and j, and these quadruplets were what he had been looking for all that time.

Now having assumed "not only $i^2 = -1$ and $j^2 = -1$, but also $k^2 = -1$, and $ij = k$, $ji = -k$," as he mentioned in the last line of the quote on p. 63, continuing the *Evening entry* Hamilton described the contemplations following the eureka moment, or the "sparks" as he later would call them in the *Letter to Tait*,

> I then thought it likely that it might be proper to assume $ik = -j$, because $ik = iij$ and $i^2 = -1$. If so, since $ji = -ij$, it seemed likely that $ki = -ik = j$ which might also follow from $k = -ji$. In like manner, it appeared possible (or at least natural to be assumed) that $kj = ijj = -i$, $jk = -jji = i$. The multiplication-assumptions, or definitions, were therefore collected to be $i^2 = j^2 = k^2 = -1$; $ij = k$, $jk = i$, $ki = j$; $ji = -k$, $kj = -i$, $ik = -j$ [4, 104].[47]

He wrote these equations in his pocket-book, and very unphilosophically, cut the fundamental formula, $i^2 = j^2 = k^2 = ijk = -1$, on a stone of the bridge.

5.4. **The second part of the walk.** After Hamilton had cut the formula on Broome Bridge, they walked to the cab stand in the "neighbourhood of the turnpike," as Hamilton mentioned in *Book C., 1848* [2, 437].[48] The turnpike was near the Cross Guns

[47] From *Carl Friedrich Gauss* Werke, Band VIII, 1900, *Mutationen des Raumes* pp. 358, 359, it is obvious that Gauss found for a multiplication of his Scalen the same 16 terms as the ones Hamilton wrote on the lower pocket-book page [gallica.bnf.fr]. Gauss' Scalen are antisymmetric to the quaternions; they would obey $i^2 = j^2 = k^2 = kji = -1$, therefore, the only difference is that the Scalen would yield a left-handed coordinate system instead of a right-handed one. But Gauss did not publish it, and indeed did not develop it into an algebra, see footnote 41.

[48] After the Hamilton Walk of 2015, Tony O'Farrell, the initiator of the annual walk, spoke about the turnpike not having been at Broome Bridge but at the next bridge, and about the "hackney stand". A hackney was "any carriage plying for hire" [britannica.com]. Because Hamilton wrote that he was sitting "on" a car, it may have been an open carriage, or a half-open one [en.wikipedia.org]. The turnpike can be seen on Historic Maps of Dublin, 1800-1849 [dublinhistoricmaps.ie], choose '1832 County of City,

Bridge, then the next bridge,[49] and because Hamilton later called the day of discovery "that to me memorable Monday"[2, 437], it can be imagined how delighted they were when walking the two kilometres between the bridges. It is not at all unlikely that during that walk they came up with the name 'quaternion' together; Hamilton had not named earlier discoveries, 'systems of rays' and 'conical refraction' were just what they are. Moreover, Hamilton never said much about what motivated him to call them 'quaternions', it seems that the only time he gave something of an explanation was a rather general remark in a footnote of his second book, *Elements of Quaternions*, "As to the mere word, Quaternion, it signifies primarily (as is well known), like its Latin original, "Quaternio," or the Greek noun $\tau\varepsilon\tau\rho\alpha\kappa\tau\acute{\upsilon}\varsigma$, a Set of Four: but it is obviously used here, and elsewhere in the present work, in a technical sense" [8, 111].

Yet they both were deeply religious, and knowing they read the Bible together, spoke with each other about religious beliefs, and prayed together [2, 311-312], it may have been more than a coincidence that the word 'quaternion' is in the Bible, The Acts 12:4, when the apostle Peter was delivered "to foure quaternions of souldiers."[50] And if perhaps, in happy excitement and seriousness not Hamilton, but Lady Hamilton, came up with the word 'quaternion', she may have asked him to keep that out of his public writings, knowing that he was wont to always give credit to whom credit was due [2, 464], [3, 76, 150]. Before accepting his marriage proposal she had asked him for a retired life, separate from his scientific, public life, and mentioning her as the name giver would certainly have gone too far. Of course, this is just an educated guess, but it would perfectly explain Hamilton's silence, both about the second part of the walk, and about why he chose the word 'quaternion'. If this was the case, by mentioning her in the *Letter to Tait* Hamilton came close to the boundaries of his wife's wishes, but he did it in the most general way.

It is not known if, or when, Lady Hamilton parted with her husband; depending on where she was going to, she may have preferred to continue walking when, at the Cross Guns Bridge, Hamilton took a car to the Academy at Grafton Street. While sitting on the car, he checked the law of the moduli.

5.5. **The law of the moduli.** In the *Letter to John Graves*, having given the considerations about i, j, k, Hamilton wrote that his "assumptions were now completed" [5, 492], and gave equations 5.1, albeit combined into four equations. Then he wrote about the requirement he had had for the triplets, and now for the quaternions,

> I considered it essential to try whether these equations were consistent with the law of moduli, namely,
>
> $$a''^2 + b''^2 + c''^2 + d''^2 = (a^2 + b^2 + c^2 + d^2)\,(a'^2 + b'^2 + c'^2 + d'^2),$$
>
> without which consistence being verified, I should have regarded the whole speculation as a failure. Judge then of my pleasure when, after a careful examination, I found that the whole twenty-four products not involving squares in the development of the sum of the squares of the four quadrinomials destroyed each other; and that the sixteen products involving squares were just those which arise otherwise from the multiplication of $a^2 + b^2 + c^2 + d^2$ and $a'^2 + b'^2 + c'^2 + d'^2$ [5, 492].

Boundary Commission', where it can be seen that the turnpike was indeed near the "Cross Guns or Westmoreland Bridge", due north of the city centre, nearly at the top of the map.

[49] The current railway bridge near Lock 7 was not there yet; it was built ca. 1864, see Buildings of Ireland [buildingsofireland.ie].

[50] *King James Bible, 1637-38* [archive.org]. It is also in John Milton's *Paradise Lost*, book 5 verse 180 [archive.org], and it is very likely that also Lady Hamilton had read the poem, because it is known that she had read her classics [2, 16]. For additional earlier uses of the word 'quaternion', even in a mathematical context, see the blog entry of 29 August 2021 [annevanweerden.nl].

This is in complete accord with *Book C., 1848* as given by Graves [2, 437-438], in which Hamilton wrote, see also the quote on p. 61,

> The pocket-book remains [...] in which [...] I did actually pencil at the time [...] the notes [equations 5.1]. In a jolted handwriting, the same pencilled page contains these other notes [visible in the middle of the lower pocket-book page, Figure 5.3], which were inserted while I was driving on that (to me memorable) Monday from the neighbourhood of the turnpike to the Academy, as the constituents of the quaternion product of the two quaternion factors,
>
> $$a + ib + jc + kd, \text{ and } \alpha + i\beta + j\gamma + k\delta.$$
>
> The dots referred to certain destructions of double products, by additions of positives to negatives, which I was examining on the car, in order to verify a conjecture which I instantly made, namely, that the law of the moduli would be found to hold good. [...] In connexion with such cancelling of terms, I pencilled also as follows [the equations on the upper pocket-book page, Figure 5.3]; which I think was done while I was sitting in the President's Chair [2, 437-438].

Explaining what the + and − signs meant, which can be seen on both pages of the pocket-book, Hamilton wrote that there were "24 (double) products, to be cancelled, 12 by 12 others" [2, 439], which would leave only "sixteen products involving squares," and that would show that quaternions obey the law of the moduli [5, 492]. Yet in the president's chair Hamilton wrote out only 9 cancelled products, as seen on the upper page of the pocket-book, and remarked, "But 3 were already known to be cancelled by 3, namely, those which did not involve d nor δ" [2, 439]. Setting $d = 0$, $\delta = 0$ [4, 105], would give a calculation similar to the multiplication of the triplets having their factor lines in one common plane with the unit line, $(a + ib + jc + 0)(x + ib + jc + 0)$, from which Hamilton had concluded that $ij = -ji$, see the quote on p. 62.

In the *Letter to Tait* Hamilton wrote, having "felt a problem to have been at that moment solved," see the quote on p. 61,

> Less than an hour elapsed before I had asked and obtained leave of the Council of the Royal Irish Academy, of which Society I was, at that time, the President – to read at the next General Meeting a Paper on Quaternions; which I accordingly did, on November 13, 1843 [2, 436].

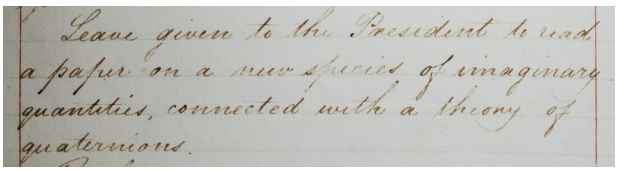
Leave given to the President to read a paper on a new species of imaginary quantities, connected with a theory of quaternions.

FIGURE 5.4. Minutes of the RIA council meeting of 16 October 1843.
"Leave given to the President to read a paper on a new species of imaginary quantities, connected with a theory of quaternions."
By permission of the Royal Irish Academy © RIA

And knowing that about discoveries, Hamilton's fellow mathematician James MacCullagh more often believed he actually had made or at least suggested them [2, 463],[51]

[51] In 1833, MacCullagh had claimed conical refraction [1, 685-692], and this tendency had been the reason that Hamilton wrote the *Letter to John Graves*, and sent it under cover to Robert Graves [2, 440]. In 1852, Hamilton wrote to De Morgan, "MacCullagh [...] was constantly fancying that people were plundering his stores, which certainly were worth the robbing" [3, 331].

Hamilton doubtless did not postpone discussing the equations with him until after the meeting; the sooner the better, and in company of a witness, William Sadleir.[52]

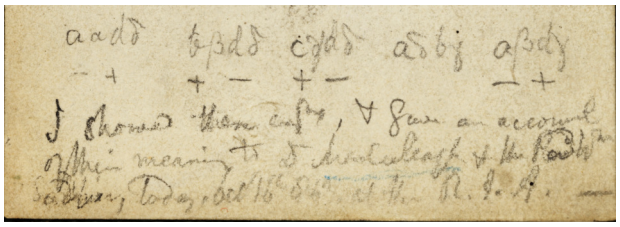

FIGURE 5.5. The lower part of the lower pocket-book page, on which Hamilton wrote, "I showed these equations, and gave an account of their meaning to Dr. MacCullagh and the Rev. William Sadleir today, October 16, 1843, at the R. I. A." With his blue pencil, Graves underlined MacCullagh's name. *Courtesy of the Board of Trinity College, the University of Dublin.*

5.6. **Chronologicality.** Hamilton's versions of the discovery containing moments in real life are found in *Book C., 1848*, the *Letter to Tait*, and the *Letter to Archibald*. There are some small differences, but as regards the sequence of events they are in complete agreement. Combining Hamilton's remark in the *Letter to Tait*, that "less than an hour elapsed" between the eureka moment and having asked and obtained leave at the Academy, and the known time of the start of the meeting, 15h30, a possible time line can be made of that afternoon. It can be used to scrutinise the credibility of Hamilton's account, and approximate the time of discovery.

What Hamilton described is that he was walking along the Royal Canal when his wife joined him. Just when they "came up" to Broome Bridge, he had his eureka moment; k was a third distinct imaginary, not to be confounded with i or j, and triplets were replaced by quadruplets. The "sparks" which fell from it were the "fundamental equations between i, j, k." He "pulled out on the spot a pocket-book" and entered the fundamental equations, and at that "very moment" felt the importance. "As they passed" Broome Bridge, he cut the "fundamental formula" on a stone of the bridge. They walked to the turnpike, he got a car to Dublin on which he started to check the law of the moduli, and he sat in the president's chair when he finished his calculations.

It further is assumed that Hamilton then asked leave to read a paper in November, and thereafter showed the equations to MacCullagh and Sadleir. Having been president, it may have sufficed to ask leave from them because both men were council members, nevertheless, asking leave first could be helpful in assuring his priority in case MacCullagh would again make a claim, which he indeed did in 1844 [2, 462-464]. That was also the importance of having this exact phrase in the minutes; it would be evidence of having then and there mentioned his discovery, including the word 'quaternion'.

5.7. **A most likely timeline.** To make a timeline of that afternoon, it is necessary to have distances and estimations of walking and driving speeds. The walk from the observatory to the Longford Bridge near Ashtown[53] is about 1.5 km. From the Longford Bridge along the Royal Canal to Broome Bridge is about 2.5 km, from Broome Bridge to the Cross Guns Bridge about 2 km, and from the Cross Guns Bridge to the Academy, then at Grafton Street [2, 192], about 3 km. Hamilton was a fervent walker, and doubtless so was his wife; she apparently did not have any problem with walking to

[52]As can be seen in Figure 5.5, Hamilton seems to have written 'Sadlier', but this was William Digby Sadleir, one of the members of the committee of science, see the *Dublin Almanac* for 1843, p. 165 [babel.hathitrust.org].

[53]For the Longford Bridge, near the 10th Lock, see Buildings of Ireland [buildingsofireland.ie].

Dublin.[54] And because it was not a walk for leisure because Hamilton had to preside, it can be assumed they walked about 6 km/h. An exact speed of the cars was not found, but by 1843 mail coaches could reach a speed of 16 km/h,[55] yet the Academy building being in Dublin city centre, the last part may have been somewhat slower, therefore the average speed can be taken as 12 km/h. Finally, the meeting would start at 15h30, which leads to the following timeline.

At 13h40, Hamilton left home, and walked to the south, 15 minutes. At 13h55, Lady Hamilton caught up with her husband at the Longford Bridge where he just turned left to start walking along the canal. They walked for 25 minutes until, near Broome Bridge, Hamilton had his eureka moment at 14h20. He experienced the moment, collected the "sparks", i.e., the fundamental equations for i, j, k, and wrote the equations in his pocket-book, 5 minutes, 14h25. He cut the fundamental formula on the bridge, 2 minutes, 14h27; it was an impulse, no time to dwell on it, he had to be in time for the meeting, it may have been Lady Hamilton who kept an eye on the time.[56] They walked to the cab stand near the Cross Guns Bridge, 20 minutes, 14h47, he got on a car, 2 minutes, 14h49, was driven to the Academy, 15 minutes, 15h04. He paid the carman and entered the Academy building, 5 minutes, 15h09, finished the calculations while sitting in the president's chair, 5 minutes, 15h14, asked and obtained leave, 4 minutes, 15h18. He showed the equations to MacCullagh and Sadleir, 8 minutes, 15h26, in time to return to his chair, make the short note in the pocket-book of having shown them his equations, and start the meeting at 15h30. This gives a total of 58 minutes between the eureka moment at 14h20 and having asked leave at 15h18, indeed "less than an hour" later, completely fitting in with his version of that day.

5.8. **Happiness.** For Hamilton, the combination of being close to his wife and working on mathematics was the happiest state of being [10, 133]. Having moreover been deeply religious, regarding marriage as a holy bond in which husband and wife became one,[57] he doubtless was very thankful that she was with him, precisely at that life-changing moment. And because symbolism was nothing strange for Hamilton [2, 256], it must have been gratifying that, as he mentioned in his *Book C., 1848*, she had given him the pocket-book in which he wrote the equations for the first time.

It is also known that Lady Hamilton was proud of her husband [9, 120]; in February 1833, two months before their marriage and having been highly praised for the discovery of conical refraction, Hamilton wrote to her, "All this, I know, will give you more pleasure than it does me, though I do not pretend that it gives me none; but I amuse myself sometimes imagining the delight with which you will open hereafter my presents from Europe and America, while I shall put on the Stoic, and tell you that I do not care for such things" [10, 156]. Yet because as a couple, they usually did not write letters to each other, there are no private letters about what she felt when her husband finally concluded the searches for triplets about which they had talked so often, leading to their sons asking about triplet multiplication during the breakfasts in October.[58] And although friends of Hamilton testified that he could not lie [10, 40], he was very well able to keep silent about something [10, 281], in this case about their happy walk between the bridges and, who knows, about how the quaternions received their name.

[54]It was mentioned before that Lady Hamilton was frequently ill, implying a 'weak health', yet it is known from Graves' biography that she also rode a horse, see for instance [1, 26-27], [3, 162, 498].

[55]See for instance *Mail coach* [en.wikipedia.org].

[56]Hamilton more often just hurried after her when he was absorbed in something else [10, 354].

[57]*King James Bible, 1637-38*, Mark 10:8, "And they twain shall be one flesh : so then they are no more twain, but one flesh" [archive.org].

[58]It can easily be assumed that she joined in these conversations; there are indications that she had a quite technical mind, [10, 152, 182, 437], although due to their social circumstances undeveloped.

6. The creation of the quaternions; Hankins' reconstruction

In the 22nd section of his biography, called 'The Creation of Quaternions',[59] Hankins gave his own reconstruction of the discovery [9, 293-301],[60] in which his general doubts about Hamilton show very clearly. He changed quite some details of Hamilton's own version, of which the most unimportant one seems to be that Lady Hamilton had not "talked with him now and then," as Hamilton wrote, but "he had been talking to her" [9, 299]. But details make up a story, and comparing Hankins' reconstruction with that of Hamilton shows that Hankins also changed chronology, and even motivations.

To be able to compare and discuss where Hankins' reconstruction deviated from Hamilton's reports, quite some quotes from his 'quaternion section' will be given in the order in which he wrote them, each time followed by comments. Paraphrasing appeared to obscure too much of their essence, and although the collection of quotes is not meant to be a complete overview of the section, it will show that Hankins' confusion may have followed from the erroneous assumption that Hamilton had been walking in the morning instead of in the afternoon, and that he therefore took a large part of Hamilton's course of thought in the morning, as described in the *Evening entry*, as having happened during the walk. It led to his conclusion that Hamilton did not yet have a complete theory when he reached Broome Bridge and, consequently, in Hankins' reconstruction there was no eureka moment, just a sudden realisation as an intermediate moment.

6.1. **Quotes from the 'quaternion section', with comments.**

1.) Referring to the 1865 *Letter to Archibald* [2, 434], quoted on p. 59, Hankins wrote that Hamilton, having returned from the meeting of the British Association in Cork, had started a new search for the laws of multiplication of triplets.

> Every morning during the month of October, he tells us, he would come downstairs to breakfast and would be asked by his elder son, "Well, Papa, can you multiply triplets?" and every morning he had to confess that he could only add and subtract them. This episode has the characteristics of a charming, but unreliable, anecdote. Even if it is false in detail, however, it indicates that Hamilton was intensely engaged again in thinking about the triplets [9, 291].

Comments. Hamilton wrote the letter to his son Archibald, who had been eight years old at the time, which makes it perfectly possible that he remembered it well. It had been a memorable event; the discovery had such an impact on the family that thereafter the boys called Broome Bridge the 'Quaternion Bridge' [2, 435]. Knowing about Hamilton's astounding memory [1, 128], [2, 457-458], [3, 153] and that even until his last days he was working on his second book, *Elements of Quaternions*, calling Hamilton's memories as related to his son an "unreliable anecdote" should at least have been accompanied by arguments, but Hankins did not give any. An erroneous detail is that Hankins wrote that the "elder son", the then nine year old William Edwin, asked the question, although Hamilton explicitly mentioned both of his sons.[61]

2.) Having considered reasons for Hamilton to have returned to the problem of the triplets, Hankins wrote about the moment of discovery,

---

[59] Next to, perhaps, a philosophical stance, Hankins' choice of the word 'creation' may have been a reaction to John Graves, who had written to Hamilton, "I have not yet any clear view as to the extent to which we are at liberty arbitrarily to create imaginaries, and to endow them with supernatural properties" [9, 300].

[60] Hankins called other articles about Hamilton's discovery "modern reconstructions" [9, 441], and it therefore seemed appropriate to use this expression here.

[61] His daughter Helen had turned three in August.

> On the sixteenth of October, 1843, he and his wife were walking into Dublin along the Royal Canal to attend a council meeting of the Royal Irish Academy. Suddenly a possible resolution of the problem of triplets leapt into his mind [9, 293].

Comments. Hankins' use of the word "possible" is curious because, in the *Letter to Tait*, see p. 61, Hamilton had written that "the quaternions started into life, or light, full grown," which implies that he was immediately certain about the quaternions. That is in accord with having written, in *Book C., 1848* [2, 437], that after the discovery, he had "verified a conjecture which I instantly made, namely, that the law of the moduli would be found to hold good," see p. 66. Even if Hankins used the word "possible" in the sense of "acceptable," it conveys doubt, for which he did not give arguments.

3.) Hankins then gave a part of the quote from the *Letter to Archibald*, p. 60. He ended the quote with Hamilton's sentence,

> Nor could I resist the impulse – unphilosophical as it may have been – to cut with a knife on a stone of Brougham Bridge, as we passed it, the fundamental formula [9, 293].

Hankins did not repeat the last part of the quote, in which Hamilton wrote out the "fundamental formula," but remarked in an endnote,

> Hamilton claimed to have written $i^2 = j^2 = k^2 = ijk = -1$ on the bridge, but it seems unlikely that the inspiration came to him in such a neat and condensed form [9, 441].

To be able to comment on this quote, it has to be combined with the following quote from Hankins. Having given a large part of the quote from the 1858 *Letter to Tait*, p. 61, in which Hamilton wrote that he had found the "fundamental equations between i, j, k; exactly such as I have used them ever since" [9, 293], Hankins commented,

> The pocket book remains to verify this story, which casts doubt on his later claim to have scratched the quaternion formulas on the bridge. Hamilton would scribble on any handy surface if he did not have paper along, but on this occasion he did have a notebook. Maybe he felt that the solemnity of the moment required an inscription in stone [9, 294].

Comments. The latter two quotes introduce Hankins' until then unexpressed supposition, that at the bridge Hamilton did not yet have a "full grown" system as he had written in the *Letter to Tait*, p. 61; Hankins did not repeat this part of the letter. But the part of the letter Hankins did give [9, 293], also contained Hamilton's remarks about having written the "fundamental equations between i, j, k" in his pocket-book, see equation 5.1, and Figure 5.3, and Hankins did not challenge that. Yet going from these equations to the "fundamental formula" $i^2 = j^2 = k^2 = ijk = -1$ is simple; if $ij = k$ and $k^2 = -1$, then $ijk = -1$. The born mathematical generalist that Hamilton was [3, 200-201, 286], and knowing how focused he then was, he could even have seen it while cutting the first part of the fundamental formula on the bridge; he would find new "tracks of discovery" while lecturing on a blackboard [10, 342]. Hankins' disbelief, which even led him to change Hamilton's word "formula" for what he wrote on the bridge into the plural "formulas", thus shows that he saw the writing on the bridge as an intermediate moment, not as an expression of a eureka moment. And suggesting that Hamilton's motive to cut formulae on the bridge was that he may have felt that the "solemnity" "required an inscription in stone," entirely passes over Hamilton's sentence, that he could not "resist the impulse."

4.) Hankins continued,

> After the discovery flashed across Hamilton's mind he quickly elaborated it while riding in a car the rest of the way to the council meeting. Once at the meeting he showed his discovery to [James] MacCullagh and to William Sadlier [9, 294].

Comments. This is a very clear statement; having to "elaborate" the discovery indicates again that Hamilton did not yet have a finished theory at the bridge. A small detail is that Hamilton wrote that he was sitting "on" a car, not "in" one [2, 437].

5.) About the sources for his section, Hankins wrote,

> [The] letter to Graves, a memorandum made the day of the discovery, and a later preface to his *Lectures on Quaternions* (1853) all contain attempts by Hamilton to reconstruct the train of thought that led him to his discovery. These documents make the moment of truth on the bridge in Dublin one of the best-documented discoveries in the history of mathematics [9, 295].

Comments. Concluding from the endnotes, Hankins gave references to Graves' biography, the *Evening entry*, and the *Letter to John Graves*, but not to the *Preface*. He also gave a reference to the paper for which Hamilton had asked leave, p. 66, [4, 111-116], but did not quote from it. His assumption that Hamilton made his discovery "on" the bridge, suggests that he did not know about the turnpike, nor about Seán Keating's charcoal drawing of the Hamiltons on the tow-path under the bridge.[62]

6.) Hankins then started to describe the walk anew, now embedded in mathematical contemplations.

> [Not finding a triplet algebra] was a problem that he had been turning over in his head and struggling with for some time prior to his discovery by the bank of the canal. What occurred to him there was the sudden realization that the "obvious" system of triplets that he had under consideration at the time would succeed if extended to a quaternion of four numbers [9, 295].

Comments. This time Hankins described what according to him Hamilton's "sudden realisation" was, in quote 2.), p. 69, only having mentioned that Hamilton "suddenly" had seen a possible resolution. So far, it does not contradict Hamilton's reports.

7.) Then Hankins gave Hamilton's "train of thought" about triplets which had led to his "sudden realization", and remarked that the square of a triplet

> should be another triplet, while the product that Hamilton obtained had four terms. He had to do something with the ij term – either set it equal to zero, or add it somehow to one of the other three terms. Hamilton thought he might find an answer in a geometrical analogy between triplets and complex numbers [9, 295].

[62] *Sir William Hamilton at The Royal Canal Dublin* by Seán Keating [sirwilliamorpen.com]. In accord with the state of the gossip then, Lady Hamilton is not happy, and Hamilton has to peek in his pocket-book in order to write the formula on the bridge. Incidentally, the initiators of the first plaque on Broome Bridge must have been right, as also Daniel Doyle acknowledged, choosing that side of the bridge for his sand sculpture, Figure 5.2; if the Hamiltons "came up" to the bridge, Hamilton doubtless cut the formula on the observatory side of the bridge [webhomes.maths.ed.ac.uk]. That first plaque was, in 1958, unveiled by the mathematician and politician Éamon de Valera, who at that time was Taoiseach. In 2009, the then damaged plaque was restored and relocated to a place higher up on the bridge [archive.maths.nuim.ie], and in 2021 it was accompanied by a sculpture [mckeanesculptor.ie].

Comments. That Hamilton "thought he might find an answer" in a geometrical analogy between triplets and complex numbers is curious; it had been Hamilton's basic assumption. In the *Letter to John Graves* he wrote, "You know that I have long wished [...] to possess a Theory of Triplets, analogous to my published Theory of Couplets, and also to Mr. Warren's geometrical representation of imaginary quantities" [5, 490].

8.) Next giving the equivalent laws of the moduli and of the norms, Hankins wrote,

> Hamilton was particularly anxious that the law of the norms hold for triplets as well as for number couples, because it would determine the length of the product line in the geometrical interpretation of triplets and would guarantee that division of one triplet by another would always give a unique quotient [9, 296-297].

Comments. Hankins chose to call the 'law of the moduli', the 'law of the norms' when Hamilton was calculating squares, even though Hamilton did not make such an explicit distinction [5, 10-11]; taking norms instead of moduli just makes it slightly easier to write down. Having called the length of a quaternion, μ, its "modulus" [5, 12], Hamilton's preference for the word 'modulus' points to his, what Graves called, "immediate prevision of the applicability of quaternions to physical phenomena" [2, 439].

9.) After discussing further steps made by Hamilton, Hankins continued that Hamilton's assumption of $ij = -ji$ did not remove the ij term from the general product of two triplets, and neither could the ij term be set to zero,

> because the product of the norms is the sum of four squares, not of three squares. But the sum of four squares cannot be the modulus of a triplet. The only thing to do was to set ij equal to some unknown k and see if it might somehow be removed [9, 298].

Comments. In both the *Evening entry* and the *Letter to John Graves*, Hamilton wrote that concluding that ij could not be equal to zero, that the $(bz - cy)^2$ term could not be neglected, led to k as a new imaginary, of which, as Hamilton wrote in the *Evening entry*, the square still could be equal to 1 or -1. That Hankins suggested that after Hamilton knew that the ij term could not be zero, he still thought he could remove it somehow, may have been connected to his remark in 7.), p. 71, that it could "somehow be added to one of the other three terms," but that consideration was not found in Hamilton's reports. Or, it comes from the first part of the *Evening entry*, in which Hamilton wrote about a new imaginary k, and then still contemplated that ij could be zero. But that was the part he thereafter corrected, as discussed on p. 62.

10.) Hankins continued,

> Hamilton had probably reached this far in his calculations before he set forth on his walk along the Royal Canal. He was musing over his triplets and their unfortunate inclination to make a quaternion of terms in the product, talking to his wife, but with the triplets in the back of his mind. Suddenly it occurred to him that if the product of two triplets had four terms and if the product of their norms also produced a norm with four terms, maybe he would have better luck multiplying expressions with four terms than he had had multiplying triplets. If k were taken to be a third imaginary in addition to i and j, his elements would become quaternions and maybe they would work better. He thought that possibly $k^2 = 1$, but could see immediately that this would not work, because some of the cross terms in the calculation of the norm would not cancel out. They would cancel, however, if $k^2 = -1$ [9, 298-299].

Comments. It is now clear why in Hankins' reconstruction there is no eureka moment; this "sudden occurrence" only yielded quaternions which might "work better." Hankins did not give references for his reasonings about three and four terms, yet Hamilton did mention it in his 1853 *Preface*, when he wrote that he saw, when multiplying triplets, "that although the product of the sums of squares of the constituents of the two factors could not in general be decomposed into three squares of rational functions of them, yet it could be generally presented as the sum of four such squares" [7, (45)-(46)], [4, 143-144], and that that was what had led him to quaternions. That is of course true, also in the *Preface* the fourth squared k term, $(bz - cy)^2$, was given as having been followed by the discovery, but this is the only time Hamilton mentioned the number of terms as a basic consideration. There are some more differences between the *Preface* and the other sources, such as giving the fundamental equations between i, j, k and the conclusion $k^2 = -1$ in reversed order [7, (46)], yet it was not contradictory, this was not a chronological report. In the *Preface* Hamilton was very clear however about having known the basic geometrical interpretations before he asked leave at the meeting [7, (47)-(48)]; in the *Letter to John Graves* he had also given geometrical contemplations, but there it was without a time indication. It is very well possible that this was one of the subjects Hamilton thought or even talked about during the second part of the walk; it may not have taken him much time to translate it from triplets to quaternions.

11.) Hankins continued,

> Although Hamilton does not tell us what in particular caused him suddenly to think of quaternions in place of triplets, I think it was probably the realization that the right members of equations 22.5 and 22.6 [the general product of two triplets, and the multiplication of their moduli, respectively, [9, 297]] contained four related terms. If the right member of 22.6 was the norm of the right member of 22.5, then the right member of 22.5 would have a conjugate quaternion analogous to the conjugate of a complex number. It is an easy matter to check, and Hamilton, who had great powers of mental calculation and who had been mulling over the calculation in his mind for years, probably did it in his head. Having already assumed $i^2 = j^2 = -1$ and $ij = -ji = k$, he got
>
> $$(a + ib + jc + kd)(a - ib - jc - kd) = a^2 + b^2 + c^2 + d^2(-k^2) - bd(ik + ki) - cd(jk + kj).$$
>
> The expected norm is $a^2 + b^2 + c^2 + d^2$, and it is obviously obtained by setting $k^2 = -1$ and $ik = -ki$, $jk = -kj$. These substitutions are entirely analogous to the values that Hamilton had already obtained for i^2, j^2, and ij. The symmetry of the equations for multiplication of imaginaries must have been a powerful impetus in favor of quaternions [9, 299].

Comments. Hankins thus assumed that Hamilton had found $k^2 = -1$ by calculating the norm, as he had stated in 10.), using a quaternion and its conjugate, as he argued here, and instead of that directly leading to the eureka moment and the "sparks" following it, the fundamental equations between i, j, k, it was just another intermediate step. But not in any of the reports did Hamilton refer to having found $k^2 = -1$ by calculating the norm using a conjugate, nor that he thus was led to $ik = -ki$, $jk = -kj$ as yet another intermediate step. Hamilton wrote that $k^2 = -1$ followed from the non-cancellation of the term $2ad\alpha\delta$ in the multiplication of two quadruplets, see p. 63, and when he did mention a conjugate, in the *Proceedings* of 13 November 1843, it was only as a statement [6, 429].

12.) Hankins continued,

> The next step was to check to see if the law of the norms held for quaternions. In order to do this he needed values for jk and ik. (He had already assumed $ij = k$.) He decided that $ik = -j$, because $ik = i(ij) = (ii)j = (-1)j$. And in like fashion probably $kj = -i$, because $kj = (ij)j = i(jj) = i(-1)$. Then summarizing his "multiplication assumptions," as he called them:
>
> $$\begin{gathered} i^2 = j^2 = k^2 = -1 \\ ij = -ji = k \qquad jk = -kj = i \qquad ki = -ik = j \end{gathered} \tag{6.1}$$
>
> These are the equations that he scratched on the bridge in great excitement. They would preserve his priority at least until he could announce his discovery that afternoon at the academy. On the way into the council meeting he checked the law of the modulus, writing out the terms in his pocket notebook. He discovered that the law holds for quaternions [9, 299-300].

Comment That Hankins suggested that Hamilton needed values for jk and ik to be able to check the law of the moduli instead of therewith rounding off the discovery, again shows that he did not believe Hamilton, who had written that the quaternions came into life "full grown," that the relations between i, j, k were the "sparks" following the discovery, that he had instantly conjectured that the law of the moduli would hold, and that he had cut the fundamental formula, $i^2 = j^2 = k^2 = ijk = -1$, on the bridge. In Hankins' reconstruction there was no eureka moment, no "sparks", no fundamental formula on the bridge, just continued calculations.
And two inconsistencies emerge in this last quote. Contemplating Hamilton's motive to cut the "equations" (6.1) on the bridge, Hankins had suggested that "maybe he felt that the solemnity of the moment required an inscription in stone," see 3.), p. 70. But this time he claimed that Hamilton had scratched them on the bridge "in great excitement," that they would preserve his priority; these two interpretations suggest a different stage of discovery. The same holds for the sentence in quote 4.), p. 71; "after the discovery flashed across Hamilton's mind he quickly elaborated it while riding in a car" [9, 294], suggests a different stage of discovery than what Hankins wrote here, that "on the way into the council meeting he checked the law of the modulus."

6.2. **Conclusion.** Doubting Hamilton's claims so explicitly is in accord with the Hamilton who emerges from Hankins' biography; a man who would openly sacrifice truth for romance. Who only loved one woman, whom he had met in his youth, and who, having lost her, "remained on the edge of alcoholism" from 1848 until the end of his life [9, 224, 347]. Hankins should at least have given arguments for his doubts, and presented them in a logical and consistent manner; in accusing Hamilton of exaggeration about what he wrote on the bridge it is not enough to write in an endnote that it was "unlikely."

Yet also in general it was hardly ever found in Hankins' biography that, when he gave his opinions about Hamilton, he explained how he had come to his conclusions. Which in turn is in accord with the observation that ever since Graves' biography, anyone can claim anything about Hamilton, as long as it fits the depressed and alcoholic view of him which had already developed during the first half of the twentieth century, and is still worsening by the decade.

7. Metaphysics, and applying mathematics

In his reports of the discovery of the quaternions, Hamilton mentioned that the 'law of the moduli' was important, yet not a single remark was found about why he wanted

his system to obey this law, or, for that matter, that he wanted it to be applicable for physics. Searching in the sources used here, only one sentence was found in which he said something about his motives; in 1852 he wrote to De Morgan, "I wished for such a modulus, as a child might wish for the moon" [3, 429-430].

But closing his 'quaternion section', Hankins wrote that Hamilton's "quest for the triplets" had been "extremely metaphysical [...]. Both he and [John] Graves combined metaphysical and mathematical concepts in an extraordinary way in their correspondence. Hamilton was convinced that the operations of the mind, including mathematical operations, are in perfect harmony with the operations of the physical world as God created it. To understand the mind is to understand the world, and vice versa" [9, 301].

Indeed, in the section before the 'quaternion section', Hankins had discussed how, following the philosopher and poet Samuel Coleridge in "the polar and triadic arrangement of nature," in 1831 Hamilton had been led to the metaphysics of polarity of power and existence [9, 285-286]. As a reference, Hankins gave a memorandum by Hamilton, about Coleridge and metaphysics, in which Hamilton remarked, "power can be manifested only by its effects, that is, by overcoming Resistance, which is Contrary Power. Existence is manifested by the struggle between two opposite tendencies" [1, 438-439].

This can be recognised in Graves' description of how Hamilton found the quaternions; he was very clear about how Hamilton connected his metaphysics to the real world. When introducing parts of the *Evening entry*, he wrote, "Thinking it of importance to put on record in this volume how immediate was his prevision of the applicability of quaternions to physical phenomena I transcribe the following passages" [2, 439].

"October 16th 1843. ... Every line of length unity is thus a geometrical mean between $+1$ and -1, and every line of length μ is a mean between $+\mu$ and $-\mu$, the line being here treated as a mere imaginary, but $\pm\mu$ as pure reals opposite to each other. And $\sqrt{-1}$ has in this view infinitely many possible positions." "Perhaps we might give a semi-metaphysical interpretation to this result by remarking that any quality itself unlocal, such as heat may be, is indifferently related to any one direction and to its opposite. This conception is very vague, but it seems to me not foreign from the subject." "In the quaternion (v, x, y, z), xyz may determine direction and intensity; while v may determine the quantity of some agent such as electricity. x, y, z are electrically polarised, v electrically unpolarised. ... The Calculus of Quaternions may turn out to be a Calculus of Polarities."

It can therefore be concluded that, for Hamilton, metaphysical reasonings were as straightforward as any physical reasoning could be. And although he thus did not explicitly mention that he wanted his triplet turned quaternion system to be applicable in physics, it is very obvious from what he did; already in the *Evening entry*, Hamilton wrote about points in space and about lines, angular distances, "coordinates of the end of the line," Warren's principles, and about "a desired agreement between the results of an algebraical process [...] and a geometrical interpretation" [4, 103-104].

8. An amazing memory, and humility

That Hamilton checked the law of the moduli while sitting on the car, and in the presidents' chair, by just placing dots and signs, is one of the many manifestations of Hamilton's astounding memory, which was very fit for, and highly trained by, his habit of calculating in his head. Next to acknowledging the psychological discovery of 1832, after which Hamilton fell in love with Helen Bayly, this amazing memory is of paramount importance in understanding who Hamilton was, because he used it for his private life as well.

How impressive Hamilton's memory was is already known from the first volume of Graves' biography; Graves wrote that when only five years old, Hamilton was "loving to

recite Dryden, Collins, Milton, and Homer" [1, 47]. In 1813, eight years old and having met the "calculating boy" Zerah Colburn,[63] he "liked to perform long operations in Arithmetic in [his] mind; extracting the square and cube root, and everything that related to the properties of numbers" [1, 111]. Hamilton never lost this ability, as can be seen from the fact that even in his one-but-last year he learned a "new and ingenious" way of "contracted multiplication" from a Mr. Boyers, a self-taught arithmetician who was employed at Dunsinea; Hamilton mentioned that it "required rather more of mental calculation in passing from step to step, yet not more than would be found easy by a practised calculator" [3, 153].

That Hamilton also used his highly trained memory for his daily life was completely acknowledged by Graves, who remarked, "I may here note that to him throughout his life thoughts were events. He would remember, when he came to a particular spot in a road or field, the conversation which years before had passed there with a friend, and recall it to that friend's memory" [1, 128]. But that was overlooked by most authors of biographies and biographical sketches, and only when it became possible to search in Graves' scanned biography, hints of "where did I see that before" could be followed up. It became apparent that Hamilton more often connected events which happened many years apart, throwing new light on things he wrote or did.

For instance, in February 1825 Hamilton had written the Valentine poem for Catherine Disney, and in the poem he had promised "To lay my triumphs at thy feet" [1, 175]. In 1853, when Catherine felt that she might die soon and invited him for an interview, to finally tell him that she had been forced by her family to marry Barlow, he offered her, on his knees, his just published book *Lectures on Quaternions*, "which represented the scientific labours of my life" [9, 352], [12, 84]. That can be regarded as overly romantic, or even, as egotistic because she would never understand it, yet Hamilton simply kept his promise.

Or, when in 1832 Hamilton had written to Aubrey de Vere about having fallen in love with Helen Bayly, De Vere had answered "how intensely anxious" he would be "until I hear that you are as happy as you deserve to be" [2, 6]. More than two decades later, in 1855, when for some months opening up to De Vere about the terrible fate of Catherine Disney [10, 311], Hamilton wrote, "I have been as happy in my own marriage as I expected, and more than I deserved to be" [3, 37]. That can be read as if Hamilton felt he had not deserved to have been so happy, yet it was, foremost, a direct answer to De Vere's earlier question.

Next to the psychological discovery and Hamilton's amazing memory, Hamilton was also defined by a life-long battle against vanity. Having been extremely highly praised from his very first years, and having been surrounded by loving and supportive relatives from both sides of the family,[64] already in 1809, when Hamilton was four, uncle James in Trim was worried about his nephew becoming "spoiled by praise" [1, 34]. And when he was seventeen, his aunt Mary Hutton was critical about what she apparently saw as boasting, and "administered a lesson of humility" [1, 113].

But Graves did not recognise this ongoing battle in his adored friend [2, 284], and even contradicted what Hamilton wrote about it himself; that self-esteem and cautiousness were strong in him, but faith and veneration moderate [10, 190]. Graves argued that "competent friends" would judge that also faith and veneration were strong in Hamilton [2, 249], and he must have reacted to what he saw, a "simple, zealous, great man" [2, 286]. Yet Hamilton at times hinted at his battle against vanity, and he was even very clear about it when in 1848 he wrote to Catherine Disney, having alluded to the difficult

[63] See *Zerah Colburn, the 'mental calculator'* [annevanweerden.nl].

[64] For how large Hamilton's maternal family was, see *How Arabella Lawrence's sister Sarah introduced William Rowan Hamilton to Samuel Taylor Coleridge* [annevanweerden.nl].

times after having lost her, "Of some great sorrow I am sure that the discipline was necessary to tame, in some degree, a spirit of self-reliance, which has not yet, perhaps, been sufficiently subdued, but against which I am learning, at least, to be more and more on my guard" [2, 611]. And in a letter to a friend the next year, he wrote that he believed that he had needed that infliction to "feel and own a Master against whom I cannot defend myself" [2, 2nd fn. on 611]. That Graves saw him as "simple," may therefore have been a reason for silent happiness.

9. The need for a new biography

Hamilton's amazing memory, his psychological discovery, and his striving to remain humble, are crucial to understanding who he was. And although Graves knew about his friend's extraordinary memory, it is very unfortunate that neither he, nor Hankins, recognised the combination of these three subjects, and their influence on Hamilton's private life, because having missed it led to the most detailed and therewith thorough distortion of any reputation found in the history of science; to see who Hamilton was, all minutiae have to be scrutinised, because not any has been left untouched.

Only in our time, being able to search through scanned texts, it is possible to connect the myriads of details given in Graves' enormous biography. Hankins, writing his biography in the 1970s without the digital advantages, had to make do with the then prevalent view on Hamilton, at that time mainly due to Alexander Macfarlane, Eric Temple Bell, and Edmund Whittaker, who during the first half of the twentieth century, in an increasing series of evolving gossip, had been turning Hamilton into an unhappy, full-blown alcoholic, as was described in [11, 14-16]. Hankins did paint a more positive image of Hamilton than his contemporaries had done. Yet not explicitly contradicting these alcoholic views, not defending Helen Bayly, and introducing Catherine Disney as an absolute, all-consuming force in Hamilton's life while connecting his distress about her with alcohol, annihilated any positivity that was still left in the views on Hamilton's private life. And due to the internet, the gossip became more widespread than ever before. It appears there is only one thing that can change the distorted view on Hamilton, and that is a new biography.

Such a biography should be written by someone who is well acquainted with Ireland in the 19th century, has a thorough knowledge of mathematics and metaphysics, combined with physics and astronomy, theology, psychology and literature, and is prepared to look beyond the gossip and start anew. Such a biography would not change the way in which the mathematicians, physicists, computer scientists, engineers, etc.[65] carry on the work following from Hamilton's enormous legacy. Yet, of course depending on what will be found in still undiscussed original sources, it is expected to show the world that all that work was not done by a depressed, overly romantic, lonely and alcoholic man.

Hamilton was way ahead of his time, and in 1854 his son William Edwin wrote,

> Sir W. is surprised at his own apathy with respect to the mode in which the quaternions are regarded by the world: e.g. Mr. Carmichael told him that M. Terquem had reprinted in Paris an old investigation of Sir W.'s, and he had not yet even the curiosity to inquire what it was. So about the review of them in the Mechanics' Magazine, of which John Graves had told him. Sir W. considers this a mark of his own indifference to contemporary fame, and as arising from his conviction that his belonged to a future age entirely [2, 693].

[65] In 1850, quaternions were applied to the problem of skew bridges [2, 649]. And for instance for protein folding both Hamilton's quaternions and mechanics, or extensions thereof, can be used, see H. R. Frost, *A quaternion model for single cell transcriptomics*, 2023 [biorxiv.org], and A. Robert et al., *Resource-efficient quantum algorithm for protein folding*, 2021 [nature.com].

That age is now. Mainly due to our digital era valuing Hamilton's work differently than it was in Graves' time, or in that of Hankins, there still is very much unexplored original information about Hamilton. Letters of women, letters which were deemed too unimportant, letters which were not sent to Graves when he prepared the biography, letters which were too personal for Graves, and of which Hankins only gave sentences. Graves was ensnared by his social circumstances; Hankins did not have the digital advantages we now have. By any criterion, Hamilton is important enough to deserve a third original biography, and regain his fame in the "future age" he dreamt of.

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