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Creating Geometry Problems

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Abstract. A method of creating Olypmiad-style geometry problems is described, and is illustrated by two example problems.

1. Introduction

Over the past year, we have created geometry problems together while attending the Irish Team Selection Test training camps and other events. We have found that, although it may seem intimidating at first, the process of doing this is very accessible with some knowledge of Euclidean Geometry techniques and tools such as *Geogebra*. In this article, we will show the method by which we devised two of our problems, to 'demystify' the process of creating Olympiad Geometry problems.

To get started, we'll need some tools.

(1) *Geogebra* or another drawing software.
While not strictly necessary, using *Geogebra* can make claims much easier to spot, and much more certain. It's possible to draw new circles and lines instantly, as well as to hide unnecessary ones.
(2) A starting point.
This is a configuration on which you would like to base your problem. It could come from your imagination, another problem, or an interesting theorem.
(3) Pen and Paper.
For sketching and calculating! Used in angle and length calculations, as well as to record properties.

Now that we have our toolbox, we can begin discussing problems.

2. Problem 1

2.1. **Problem Statement.** Let triangle ABC have incentre I, incircle ω and circumcircle Ω. Let E and F be the tangency points of ω on AC and AB respectively. Choose S to be the point on Ω satisfying $\angle ASI = 90°$. Let N be the midpoint of the arc BC containing A, and P the intersection of NI with BC. See Figure 1.

Prove that AP, SI, and EF concur.

2.2. **Motivation.** We would like to note that much of the discussion in this problem and the next hinges on some knowledge of projective geometry. Some information on these techniques can be found in this handout [1]. Importantly, we will denote the cross-ratio of four points A, B, C, and D on a line or a circle, as defined in the above handout, as $(A, B; C, D)$.

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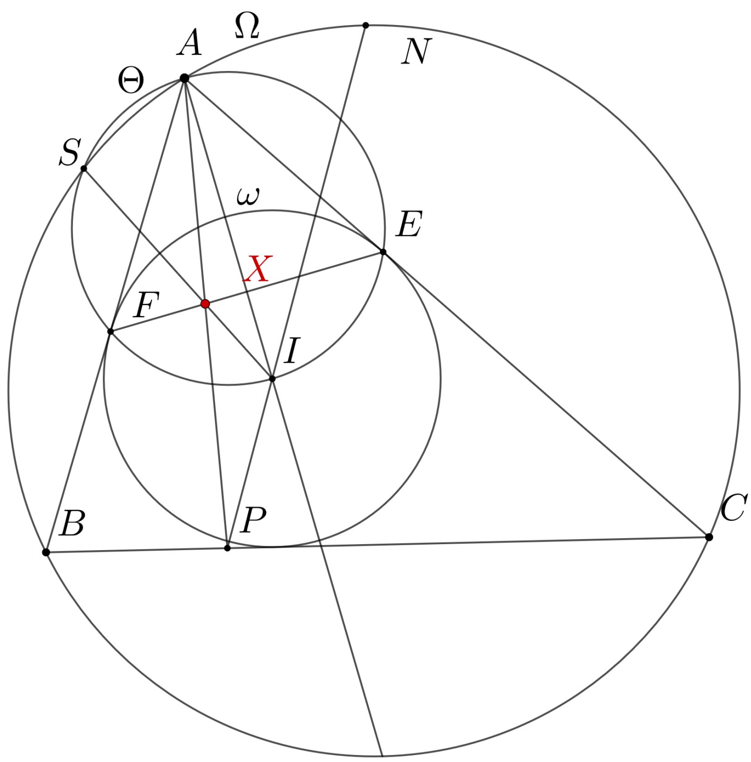

FIGURE 1. The setting of Problem 1

We started this problem with the following observation: when we take the incentre I of $\triangle ABC$, the circumcentre of $\triangle BIC$ lies on the A-angle bisector. The proof of this is left to the reader: try angle chasing. This implies that, when we take the circle of diameter AI, Θ, it will be tangent to the circumcircle of $\triangle BIC$, which we denote by Γ as shown in Figure 2.

Now we include some extra points. When we have a tangency, it seems natural to add in the tangent line. We do this and label its intersection with BC as T. By the so-called 'radical axis theorem', we know that AS, BC, and the common tangent to Θ and Γ concur at T.

We have two more 'set-up' points. We add in Q, the intersection of the other tangent from T to Γ. The reason behind this is symmetry: Γ is an important circle, and only including one tangent does not make full use of it. Our final point is N, as we defined in the initial problem statement. The motivation for adding N is the following fact: it is the intersection of the tangents to Γ at B and C. We leave the proof of this fact again to the reader.

At this point, we are able to notice something interesting in Figure 2: it seems as if N, I, and Q are collinear. Moreover, if we recall that P is the intersection of IQ and BC from the problem statement, it looks as if EF, SI, and AP concur.

We will begin by proving that N, I, and Q are collinear. This is relatively straightforward: we know that $(I, Q; B, C) = -1$, and this implies that N is on IQ as it is the intersection of the tangents to Γ at B and C.

We can also use projective techniques to prove the concurrency. Let Y be the intersection of AT and EF, and X be the intersection of SI and EF. First, $(A, I; E, F) = -1$ as $AEIF$ is a kite. Projecting through S onto EF gives $(X, Y; E, F) = (A, I; E, F) = -1$.

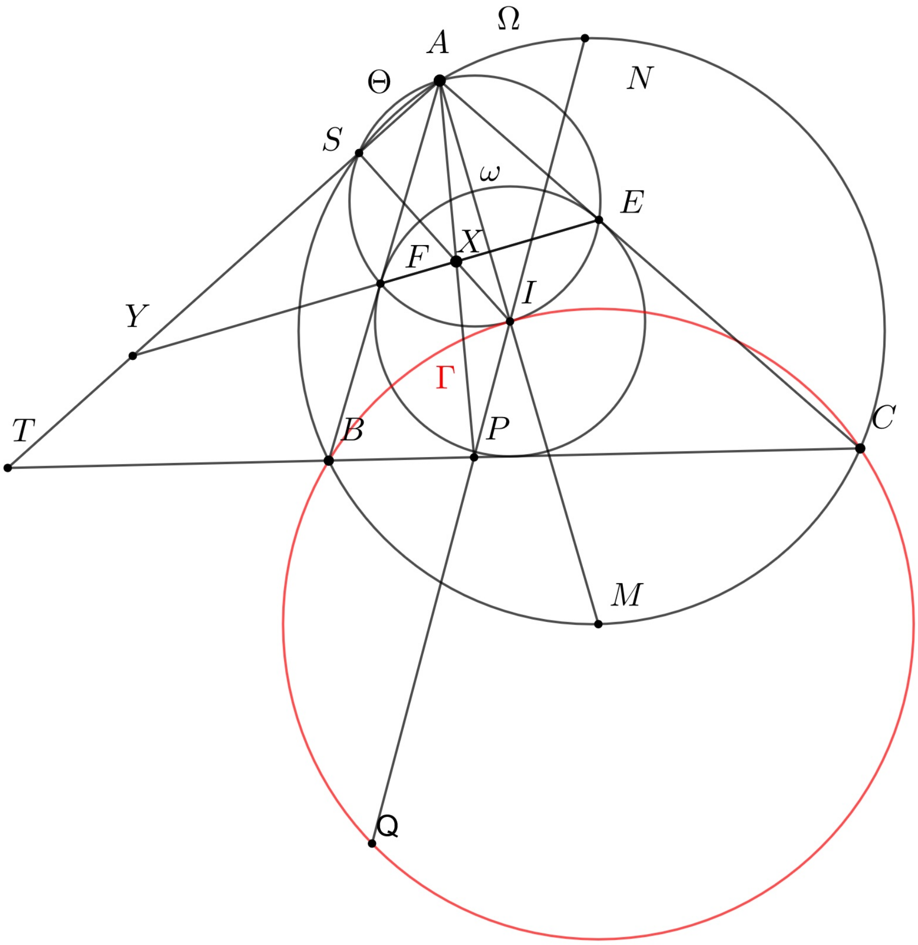

FIGURE 2. Extra circle and points for Problem 1

Now let X' be the intersection of EF and AP. Notice also that $(T, P; B, C) = -1$ and, by projecting through A onto EF, we get that $(Y, X'; E, F) = (T, P; B, C) = -1$. But this implies $X = X'$, and so we have proved the concurrency!

But what makes this problem interesting is how well we can hide this concurrency. Compare the problem statement to our final diagram above: very few of the extra points we have defined are included. This means students are challenged to 'find' the original configuration to solve the problem. The act of hiding intermediate steps can make problems more difficult and help students feel as if they are discovering the problem rather than solving it.

3. PROBLEM 2

3.1. **Problem Statement.** Let ABC be a triangle with incenter I. Define D, E, F to be the intersections of the incircle with sides BC, AC, AB respectively. Let D' be the antipode of D with respect to the incircle. Let the line parallel to BC through D' meet AB,AC at points B', C' respectively. Let the lines BC', and the line through I perpendicular to $B'C$, meet at G. See Figure 4.

Show that $\angle EGF = \frac{1}{2}\angle A$.

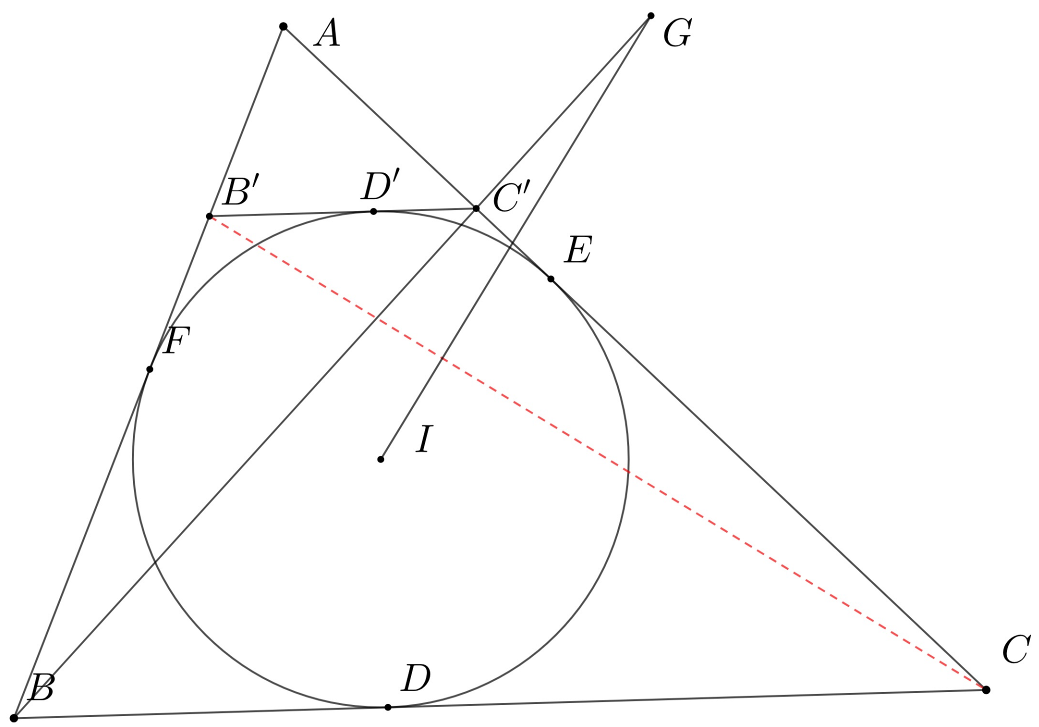

FIGURE 3. The setting of Problem 2

3.2. **Motivation.** As with most problems, This began with an interesting property noticed in a previous problem. In trying to solve one problem, one often encounters many interesting but typically irrelevant properties that end up unused in a final solution.

The source of the original problem is lost to our memory but, in attempting a solution, we began to notice some strange concurrencies. Originally we saw that lines BC', FD' and DE seemed to concur at point G. This leads to some follow up questions: are there any more lines passing through this point and why do they concur?

Following these questions, we began to play around with more possible lines. We can first note that all of our initial lines are either tangents to the incircle or directly related to tangency points of the incircle. This gives a major hint of the underlying feature at play in this problem (Poles and Polars).

Now, we have twofold motivation to consider the line through I perpendicular to $B'C$. Firstly, with enough experience using Poles and Polars, this configuration can be automatically recognised due to the number of tangencies. Secondly, we have already used line BC' in which both B and C' were intersections of tangents, hence why not consider the line through I (as every point so far can be defined using I and triangle ABC) perpendicular to the opposite of BC', CB'?

If these properties hold true we are starting to have a nice configuration! We will now confirm the concurrency of the four lines. We do this by using point redefinition: Let FD' and DE meet at G'. We will try prove that $G' = G$.

Here is where prior knowledge of Poles and Polars comes into effect. We note that the Polar of B' is FD' due to tangency and the Polar of C is DE again due to the tangency. See Figure 4.

By La Hire's Theorem we know that since G' lies on the polar of B', B' lies on the polar of G'. Applying this to C as well we get that both C and B' lie on the polar of G', and therefore CB' is the polar of G'. Hence as $B'C$ is the polar of G', the line $G'I$ must be perpendicular to $B'C$.

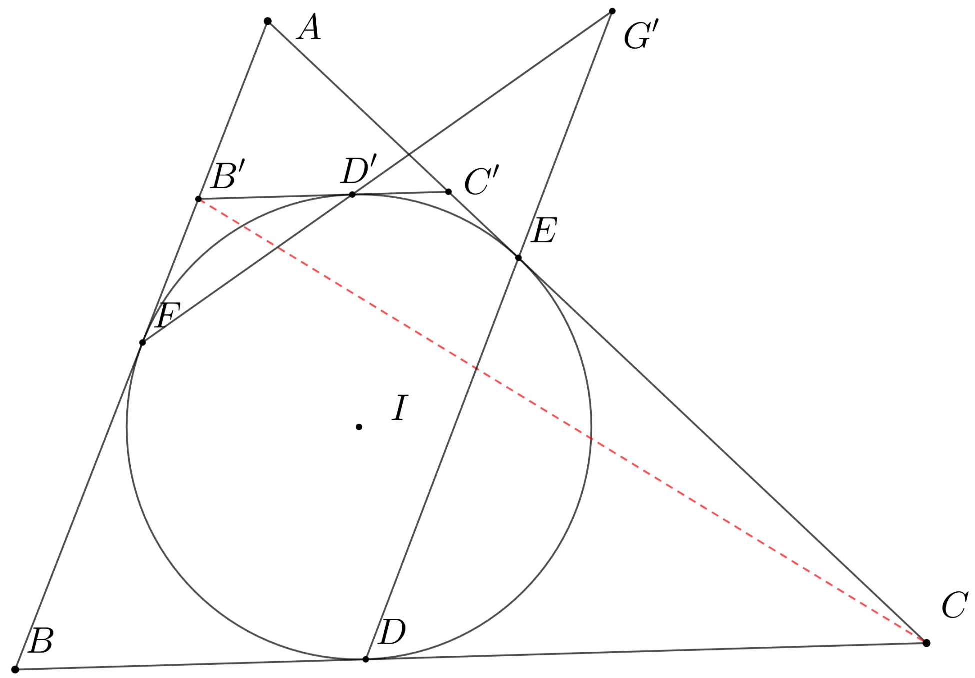

FIGURE 4. Intermediate diagram for Problem 2

Proving that G', C' and B are collinear turns out to be slightly more tricky. This can be achieved using complex numbers or Menelaus's Theorem, and it shows $G' = G$. The details of this step are left to the reader.

Again, we can note that a lot of geometry is simply seeing the obvious properties. This is a skill in and of itself and can be used in both solving and creating geometry problems. To finish our problem, we simply have

$\angle GFD = \pi/2$ as $\angle D'FD = \pi/2$ hence:

$$\angle EGF = \pi/2 - \angle FDE = \pi/2 - \angle AEF = \pi/2 - (\pi/2 - 1/2\angle A) = 1/2\angle A$$

In writing the problem statement, we try to remove as much information as possible. In this case, our simplification leads to only the intersection of two lines in the problem statement.

Much more can be done with this configuration to create other interesting problems. For example, we have the following properties: If G^* is defined as the symmetric point of G, then

- G^*, A and G are collinear;
- $G^*AG \parallel BC$;
- A is the centre of circle GG^*FE.

4. Conclusion

In making geometry problems a few things are required:

- An interesting property or initial observation;
- An understanding of why the property/properties work;
- A good diagram for proving and expanding upon the problem;
- Ideally, a clever way of concealing your initial construction.

Many observations can simply come from solving other problems in which extraneous properties are found. As was stated earlier, there are many properties even in this

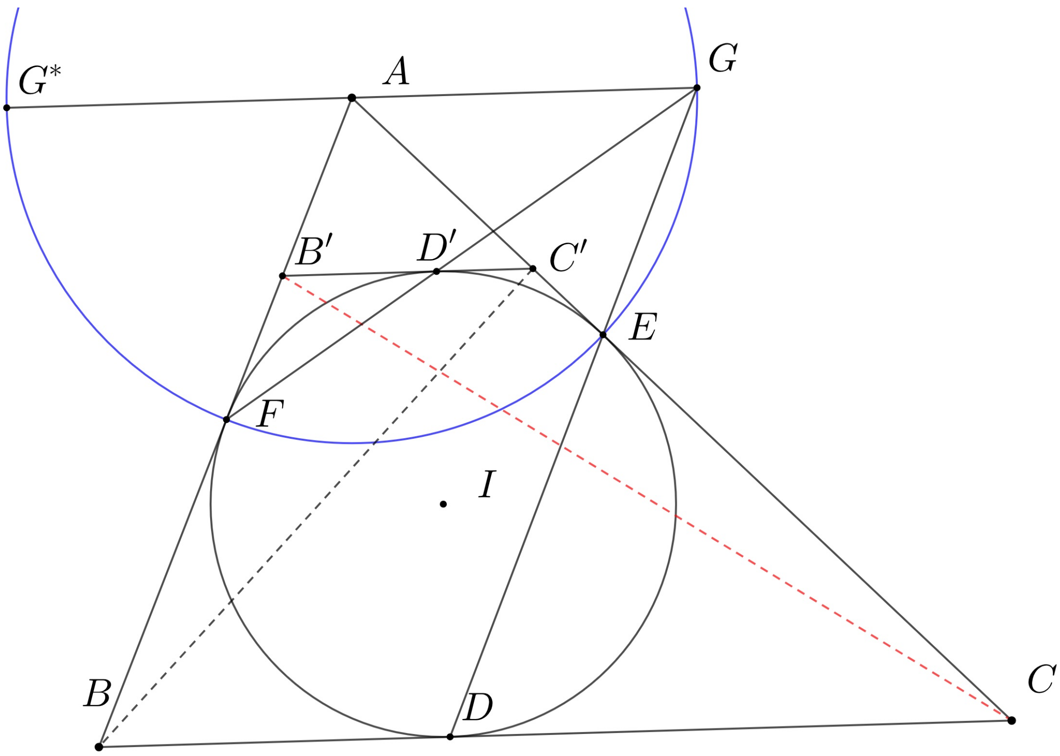

FIGURE 5. Final diagram for Problem 2

problem which were not used and as a result this problem could be expanded upon in the future.

The better you can hide or conceal the initial diagram as seen in Problem 1, the more interesting and rewarding the problem becomes.

We hope that this article has been informative and, above all, enjoyable to read. The creation of geometry problems is an art, but it is not one with an especially high barrier to entry: Hopefully, some readers will be inspired to try it out.

REFERENCES

[1] Alexander Remorov: IMO Training 2010, *Projective Geometry*. Downloaded from Projective Geometry on 19 May 2026.

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