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PROBLEMS

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The first problem was contributed by Finbarr Holland.

Problem 68.1. A polynomial is said to be *stable* if all its roots have negative real part. Suppose that

$$p(z) = a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0, \qquad a_n \neq 0,$$

is stable. Prove that

$$q(z) = a_{n-1}z^{n-1} + 2a_{n-2}z^{n-2} + \dots + (n-1)a_1z + na_0$$

is also stable.

The second problem is a classic

Problem 68.2. Let A denote the set of positive integers that do not contain a 9 in their decimal expansion. Determine whether the sum

$$\sum_{n \in A} \frac{1}{n}$$

converges or diverges.

I discovered the final problem while trying to generalise the formula

$$|AB||A \cap B| = |A||B|,$$

which holds when A and B are subgroups of a finite group.

Problem 68.3. Given subsets U and V of a finite group G, define

$$UV = \{uv : u \in U, v \in V\}$$

and

$$U^{-1} = \{ u^{-1} : u \in U \}.$$

Prove that

$$|AB||A^{-1}A \cap BB^{-1}| \ge |A||B|,$$

for any pair of subsets A and B of G.

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We invite readers to suggest their own problems, and to offer comments and solutions to past problems. In later issues we will publish solutions and acknowledge problem solvers. Please email submissions to imsproblems@gmail.com.

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