Gerard J. Murphy (1948–2006)

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1. Introduction

Following an illness that lasted for about one year, Gerard Murphy, MRIA, Associate Professor of Mathematics at University College, Cork, died on October 12, 2006, of colonic and liver cancer. What follows is an account of his life and scholarly work, that is based on information given to me by his wife, Mary, his sister, Carol, Des MacHale, David Simms, Roger Smyth, Richard Timoney, Trevor West and Stephen Wills, to whom I express thanks.

2. The Early Years

Gerard John Murphy was the first-born of Mary and Laurence Murphy, a window-cleaning contractor. He was born on November 12, 1948, and had two brothers and five sisters. The family resided in Drimnagh, Dublin 12, and Gerard and his siblings attended their local school—Our Lady of Good Counsel, Mourne Road.

Along with many other boys of his generation and social background, Gerard left school at the age of fourteen to supplement the family income, and took up his first job with the Post Office, working as a telegram-boy out of the GPO, O'Connell St. But he soon tired of this, and went to work for his father instead. But this too failed to satisfy him, and, to ease the daily drudgery, he began reading whatever books he could lay his hands on, and became a voracious reader. As luck would have it, the Simms family, who had been his father's customers since 1949, had built up a good relationship with the Murphys, and they very willingly lent Gerard books, including, in particular, a set of encyclopedias, which he absorbed. Interestingly, his brothers, who were close to him in age, also became avid readers, and later on, one studied music, and the other, art. But his sisters also were influenced by his success and love of learning, and just recently his sister Carol took her PhD in Psychology.

As time went by, Gerard became more and more disillusioned with manual labour, and decided to better himself by furthering his education. After working as a window cleaner for about five years, he decided to quit his job, brushed aside all opposition to this course of action, and proceeded to educate himself at home, with a single-minded approach that was one of his distinguishing traits. To help him achieve his goal, which was to do Engineering at TCD, he signed on with the International Correspondence Schools to prepare for A-level courses in Mathematics and Computer Science. According to Carol, he was in a welter of excitement when his first batch of study material arrived, and couldn't wait to get started! It is perhaps noteworthy, too, that, from time to time, when he was studying for A-level Mathematics, to qualify for entry to TCD, he received help from David Simms, whenever he needed it.

When he was ready to sit his A-level examinations he had to take himself to London to do them, a daunting enough task for someone who had never been outside Ireland before. He completed these successfully, and subsequently satisfied the Matriculation requirements for TCD, and applied to do Engineering there. But this was before the points system came into operation, and places in the Engineering School at TCD in those days were allocated on the basis of examination results and headmasters' reports, and, because he hadn't been to a secondary school, Gerard failed to satisfy the admission criteria. So, he couldn't do Engineering. But, by this time, he had developed a taste for Mathematics, and applied to do a degree in Honours Mathematics instead. But here, too, in not having English or another language as a matriculation subject, he fell short of the entry requirements for this programme as well, which the then Senior Lecturer wouldn't waive, despite David's protests. So, he was initially forced to register for a General Studies degree, which involved doing Mathematics and Applied Mathematics at a level well below his capabilities.

Thus, after overcoming these various hurdles, rather unusually for a future mathematical scholar of distinction, Gerard came late to the "groves of academe", and entered the portals of TCD in October 1970, a little short of his 22nd birthday, to do a BA (General) degree. But, once inside, he appears to have been allowed by the Professor of Pure Mathematics, Brian Murdoch, to attend the Special Honor Mathematics course, and take an examination in it at the end of the first term, at which he excelled, so much so that the Senior

Lecturer was persuaded to transfer him to Honours Mathematics in January, 1971. Thereafter, it was plain sailing for him. He joined the Special Honor class, which included, among others, Paul Barry (WIT), Colm Ó Dúnlaing (TCD) and Ray Ryan (NUIG), made rapid progress, and was subsequently awarded a Foundation Scholarship, which took care of his College fees, and board and lodging.

After a brilliant undergraduate career, he graduated from TCD in 1974, with a First Class Honours degree, earning a Gold Medal for the quality of his answering. Once the results of the final examination were known, he received a memorandum from Brian Murdoch, who congratulated him on his "superb performance", and noted "that it was probably the best year we have ever had in Mathematics". According to David Simms, Gerard showed an inclination for Pure Mathematics, when, while studying Synge and Griffith, he became puzzled by the way a mathematical concept was introduced!

3. Cambridge Days

Following his success at TCD, which singled him out as a special mathematical talent, Gerard was awarded a Gulbenkian Research Studentship by Churchill College, Cambridge, which he held for the next three years. This covered all his University and College fees, and, in addition, provided a maintenance allowance of 715 pounds *per annum*. Thus, in the Autumn of 1974, he was able to enroll at Churchill College, Cambridge, and study there for the degree of Doctor of Philosophy, unencumbered by financial considerations.

Non-Archimedean Banach Algebras is the title of Gerard's doctoral thesis [7], which he wrote under the guidance of Dr. G. A. Reid, and submitted in the month of April, 1977, after just two and a half years of study.

The theory of non-Archimedean Functional Analysis was begun in the 1940s, and, in the succeeding decades, efforts were made to extend the standard theorems of classical Functional Analysis by replacing the underlying field of real or complex numbers with a non-Archimedean field, namely, a field F that is equipped with a non-trivial *valuation*, i.e., a mapping $|\cdot| : F \to [0, \infty)$, that assumes at least three different values, that is multiplicative, and induces a complete ultrametric on F, so that

$$|x - y| \leq \max\{|x - z|, |z - y|\} \qquad (x, y, z \in F).$$

(In what follows immediately, F will denote such a field.) The standard example of such a field is provided by the p-adic numbers[1], and, no doubt, this served to motivate the study of other algebraic structures over a non-Archimedean field. (It seems to me, though, that people who investigated such concepts were, perhaps unwittingly, merely following Darwin's dictum that one should carry out a damn-fool experiment every so often, a suggestion with which J. E. Littlewood seemingly concurred [5]!)

By the early 1970s the theory of non-Archimedean Analysis had been extended to such areas as Banach Spaces, Harmonic Analysis and Complex Analysis, and two books had appeared, one in 1970 by A. F. Monna [6], and another in 1971 by L. Narici, E. Beckenstein and G. Bachmann [20], where the basic theory of Banach algebras over a field F is worked out, and the differences between this and Gelfand's theory are highlighted. (A Banach algebra over F is an associative algebra that is endowed with a sub-multiplicative, ultrametric, complete norm.) To get an overview of the subject of non-Archimedean spaces, and the impact it has made, see also [19].

While efforts had also been made to extend the theory of C^*-algebras to a non-Archimedean setting, these were not terribly successful, apparently; and the area was ripe for further development when Gerard was admitted to Churchill College in 1974. His supervisor, Dr. Reid, set Gerard the task of developing a more satisfactory theory of these structures, a project he successfully completed, unaided, winning the Knight Research Prize in his second year of study on foot of an essay he wrote at the time.

Confining himself almost entirely to commutative C^*-algebras with a unit, Gerard obtained an appropriate analogue of the Stone–Weierstrass theorem—thereby extending Kaplansky's version of it [4]—which was an important first step, and introduced the concepts of bundles, L-algebras, Boolean spaces and idempotents into the subject. For example, he used the concept of idempotent to overcome a marked deficiency that a field F possesses, namely, it lacks the notion of a non-trivial 'conjugation-like' self-map. As a result, it wasn't clear what the 'correct' definition of a C^*-algebra should be in this new framework.

[1] I first learnt about such things from P. B. Kennedy as a fresher in UCC, but he never gave the context, and I didn't fully understand such matters until much later. However, he invariably set a question about p-adic valuations on the examination paper.

A *bundle* is simply a family of Banach algebras over F indexed by a topological space. If the latter is compact, Hausdorff and totally disconnected, we get a Boolean bundle. The notion of a bundle gives rise to that of an L-algebra on the bundle. Gerard's version of the Stone–Weierstrass theorem in a non-Archimedean setting reads: If A is a separating Banach algebra on a Boolean bundle, then it is an L-algebra on the bundle. Perhaps somewhat surprisingly, this does include the classical theorem!

As defined by Gerard in his thesis, a C^*-algebra is a Banach algebra A over a field F with the properties that all non-zero idempotents have norm 1, and each maximal ideal of A is generated by its idempotents. Thus, F itself is a C^*-algebra, as is the algebra $C(K, F)$ of continuous functions on a compact set K that take their values in F, with the supremum norm, the idempotents being the characteristic functions of the clopen subsets of K. Gerard developed a satisfactory theory of such C^*-algebras that runs parallel with the classical Gelfand theory, and, as well, discusses fully a list of some interesting examples. All of this, and more, is contained in his Ph. D. dissertation.

His first research paper [8], which appeared in 1978, contains a very readable account of the main ideas touched on above. Indeed, as far as I'm aware, this was the only paper he ever published on the topic, even though he was occupied with the theory of C^*-algebras for the rest of his life. Remarkably, too, he never mentions the subject of non-Archimedean algebras in his book [11], not even amongst the exercises. But already in this paper one can discern early signs of his ability to present difficult ideas in a clear and cogent manner, a skill which was another of his hall-marks. Aside from this, moreover, one learns from his thesis his penchant for algebraic methods and axiomatics, his sense of mathematical aesthetics, his ability to deal with abstract concepts, and his knowledge and understanding of several different areas of Algebra, Topology and Functional Analysis, skills which he displayed in abundance later in the seventy or so research papers he subsequently wrote.

4. Back in Trinity College, Dublin

Following his stint at Cambridge, Gerard returned in the Autumn of 1977 to Trinity College, Dublin, where he held a Government Postdoctoral Research Fellowship for the next three years; and also did some teaching there. There, too, he commenced an active and fruitful, but discontinuous, collaboration with Trevor West, with whom he subsequently wrote six research articles, only five of which appear to have been reviewed. (Trevor presented their joint paper "Removing the Interior of the Spectrum—Silov's Example" at the Second International Symposium in West Africa on Functional Analysis and its Applications, in Kumasi, 1979, but it was not reviewed.) Their joint paper [18] contains, *inter alia*, a formula for the spectral radius, $\rho(a)$, of an element a in a C^*-algebra[2] A, viz.,

$$\rho(a) = \inf\{||e^h a e^{-h}|| : h \in A, h = h^*\},$$

a very beautiful result for which they will both be remembered, although Trevor attributes it wholly to Gerard.[3]

The "Little Red Book", to which Gerard often referred, had its origins in discussions Roger Smyth and Trevor had in the mid 1970s, prior to Gerard's second coming to TCD, about the possibility of extending the notion of finite rank operators to Banach Algebras. Roger told me that they were inspired to select bi-ideals of algebraic elements as suitable candidates by reading [2], and were further motivated by Rien Kaashoek, whom they met in Amsterdam in 1975 on their way to Oberwolfach, who told Roger that the concepts of finite rank and Riesz elements would command far wider interest if they could be extended to take in Fredholm elements as well. Following up this suggestion, Trevor arranged for Bruce Barnes from Oregon—who had written on such matters—to come on sabbatical to TCD, and collaborate with them to develop their ideas; and, later on, the three of them were joined by Gerard. Their joint venture led to the publication in 1982 of the research monograph [1], the aim of which the authors state was "to highlight the interplay between algebras and spectral theory which emerges in any penetrating analysis of compact, Riesz and Fredholm operators on Banach spaces". According to both Trevor and Roger, Gerard's main contribution to

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[2]From now on, by a C^*-algebra is meant a Banach algebra with a *-operation satisfying the same algebraic properties as the adjoint map for Hilbert space operators plus the key property $||a^*a|| = ||a||^2$. They are the norm closed self-adjoint algebras of operators on a Hilbert space, considered up to *-isomorphism.

[3]*Editor's Note*: This paper has very recently been made good use of once again in [M. Mathieu, A. R. Sourour, Hereditary properties of spectral isometries, Arch. Math. (Basel) **82** (2004), 222–229].

the production of this book was his first-class grasp of Spectral Theory, his knowledge of, and expertise in, C^*-algebras, and his ability to solve whatever problems arose in his area of speciality during the course of their joint investigation.

5. Abroad in North America

To gain further professional experience, Gerard took up a post of Research Associate in Dalhousie University, Halifax, Canada, which was funded by a two-year fellowship from the Canadian Government. There he also lectured to undergraduates on a part-time basis. While there he came into contact with Heydar Radjavi and Peter Fillmore—who was primarily responsible for arranging his fellowship—two world-renowned mathematicians who have made significant contributions to the field of C^*-algebras.

He spent two rewarding years in Halifax, and, during the second year, married Mary O'Hanlon, formerly a nurse and midwife who worked in her professional capacity in Cork, the Channel Islands, the United States of America and Dublin, where they met for the first time. Theirs was to be a fruitful and very happy union, which was blessed with four lovely children, Alison, Adele, Neil and Elaine.

He then moved to the United States of America, and held two one-year appointments at Associate Professor level, first at New Hampshire, and then at Oregon State University, where he again linked up with Bruce Barnes. At both these places, Gerard lectured full-time to undergraduate students, and continued his research activities. During their stay in New Hampshire, Gerard and Mary renewed their marriage vows in Church, an event that was celebrated by their families.

His period in North America was also a very productive time for Gerard, and he completed at least seven papers, one of them with Radjavi, and two with C. K. Fong, who held successive positions at the Universities of Toronto and Ottawa. He was also invited to lecture on his research at other universities, such as Toronto, Illinois, Indiana and Vancouver. In this way, he spent four years in North America, gaining invaluable teaching and research experience and making important research contacts, as well as acquiring an understanding of different systems of university education, lessons which were to stand him in good stead when he returned home in 1984. He was especially impressed by the quality of the teaching he experienced there, and this motivated him to strive for excellence in his own teaching.

6. Cork, 1984–1990

Gerard returned to Ireland in 1984 to take up a permanent appointment as a Lecturer in the Department of Mathematics in University College, Cork, where he worked for the rest of his career. He taught and examined a wide range of courses, delivering every type of course at every level, from first year calculus courses to very large classes, through to advanced postgraduate courses to small groups of students. He was a versatile teacher, and as well as giving general level courses to Arts, Commerce and Science students, he taught Control Theory to Fourth Year Electrical Engineers, gave undergraduate courses on Topology, Functional Analysis and Measure Theory to Honours students, and delivered courses on Banach Algebras and Operator Theory to postgraduate students. His principal objective in teaching was to further the students' understanding and appreciation of the intellectual beauty and depth of Mathematics and its power as a tool for understanding other disciplines. He put a lot of thought and preparation into his courses, which were designed to reflect his own approach and ideas in terms of selection of material, examples, homework and student motivation. Never content to use a colleague's lecture notes, he always designed his own. These were models of clarity and precision, and are as fresh and novel today as they were when he delivered them.

From early on he organized weekly research seminars for his postgraduate students, postdoctoral assistants and interested staff, and exposed his own and other researcher's thoughts on contemporary results in his field of interest. While these seminars were largely for the benefit of his four PhD students[4] and other postgraduates whose theses he supervised for their Master's degree, they were very informative, and kept the rest of us abreast of current developments in his speciality. Indeed, it was at these that I, and, I'm sure, many of his former students became acquainted with such topics as noncommutative geometry and quantum groups, which occupied him for

[4] Mícheál Ó Searcóid, (Fredholm theory in rings, 1987), Tadhg Creedon (Derivations that map into the radical, 1995), Kamaledin Abodayeh (Compact topological semigroups, 1998) and Adel Bashir Badi (Index theory for generalized Toeplitz operators, 2005)

the last decade of his life. He took delight in the success of all his students, and took great interest in their subsequent careers. He was especially proud that one of his MA students, Thomas Cooney, won the prestigious NUI Traveling Studentship in Mathematics in 2002 for his thesis "Amenability and coamenability of quantum groups".

From the moment he set foot on the UCC campus, he was eager to host regular International Mathematical Conferences here, and the first one, entitled "Aspects of Analysis", co-organised by Gerard, Brian Twomey and myself, was held in mid-May, 1986. This was well-attended, and attracted experts in the fields of Operator Theory and Function Theory. Buoyed by its success, Gerard was encouraged to organise an international conference on his own on "Operator Theory and Operator Algebras" in each of the following three years. These were very well organised, immensely successful, and very popular with the participants, numbering between 30 and 40, who came from all over the world. Indeed, some people were disappointed when none was held in 1990! But, by then, he had decided to ease the burden on himself, and hold them less frequently. And, for that reason, the next one was held in 1991, to be followed by others at two-yearly intervals until they lapsed again for a period after 1995. They weren't held again until 2003.

As well as carrying out his normal every-day duties during his first seven years at UCC, and busying himself organising conferences, he found time to continue to produce a steady stream of high quality research articles on Toeplitz operators and C^*-algebras, and prepare a textbook about the latter subject for postgraduate students. So, it's only right that, at this point, I should interrupt this narrative, and try to describe his contributions in these areas.

6.1. **Toeplitz operators and Toeplitz algebras.** Beginning in 1987 Gerard wrote about 20 research papers on these topics. Indeed, his last published paper dealt with them, as does another long paper [17] which has yet to appear. It seems fitting therefore to describe briefly the salient points of a subject that occupied his attention for two-thirds of his active research life, and to mention some of his contributions to this important area.

The study of Toeplitz operators begins with Toeplitz's discovery in 1911 that if $\dots, a_{-2}, a_{-1}, a_0, a_1, a_2, \dots$ are the Fourier coefficients of a bounded function, then the bilinear form $\sum_{i,j=1}^{\infty} a_{i-j} x_i y_j$ is bounded on the sequence space ℓ_2. Much later, it was realised that the converse statement is true, and that the corresponding operator could best be investigated by treating it as an operator on the classical Hardy space H^2 consisting of square integrable functions on the unit circle whose Fourier coefficients vanish on the negative integers. While much of the early work focused on the analytical properties of such operators, Brown and Halmos [3] gave the subject a new impetus in 1963 when they showed, *inter alia*, that the class of Toeplitz operators on H^2 coincides with the commutant of the unilateral shift, itself a Toeplitz operator, and emphasised the algebraic approach, a point of view that Gerard adopted in his own investigations.

He divided his attention between two of several possible generalizations of the classical theory of Toeplitz operators. On the one hand, he considered them as compressions of bounded multiplication operators acting on $L^2(\hat{G}, m)$ to an abstract Hardy space $H^2(\hat{G})$—the subspace of functions in $L^2(\hat{G}, m)$ whose Fourier transforms vanish on $\{x \in G : x < 0\}$—where $\hat{G}$ is the Pontryagin dual of a fixed ordered abelian group G, and m is normalised Haar measure on $\hat{G}$. In other words, the objects of interest for him, here, were those operators T defined on $H^2(\hat{G})$ by $Tf = P(\phi f)$, where ϕ is bounded on $\hat{G}$, and P is the orthogonal projection of $L^2(\hat{G}, m)$ onto $H^2(\hat{G})$. In this context, much of the theory of classical Hardy spaces carries over, and, thence, that of the corresponding Toeplitz operators, but, as Gerard explains in his survey article [10], many new insights emerge about C^*-algebras in general as a result of examining the particular C^*-algebra generated by Toeplitz operators. This point of view dominated his thinking throughout his career, and was one of the reasons that motivated him in his pursuit of an abstract theory—the belief that an abundance of concrete examples would act as guiding principles in the formation of an abstract theory, which, in turn, would lead to a better understanding of the classical theory.

He formulated a different class of examples of Toeplitz operators as follows. Let A be a function algebra on a compact Hausdorff space K. Suppose m is a probability measure on K that determines a continuous multiplicative linear functional τ on A such that $\tau(f) = \int_K f\,dm,\ f \in A$. Define the Hardy space H^2 to be the norm closure of A in $L^2(K, m)$. If $\phi \in L^\infty(K)$, the Toeplitz operator with symbol ϕ is then defined by $T_\phi f = P(\phi f)$, where P is the orthogonal projection of $L^2(K, m)$ onto $H^2(K)$. (We recover the classical theory, when K is the unit circle, A is the algebra of polynomials, $\tau(f) = f(0)$, $f \in A$, and m is normalised Lebesgue measure on K.) Rather remarkably, as Gerard has ably demonstrated in a series of papers that he produced about such operators in the last twenty or so years, many of the classical results about Toeplitz operators extend to this more abstract setting. Indeed, in his review of one of Gerard's papers [12], Sheldon Axler writes: "Most of the classical results hold, although often new proofs are needed in this context. The author has come up with proofs that are clean and sometimes add new insight to the classical case. Even when the classical results fail to generalize, the author has usually found an interesting substitute. For example, in the classical case, if ϕ is real-valued and $\phi \neq 0$, then T_ϕ has no eigenvalues. This fails in the more general context, but the author shows that any eigenspace of T_ϕ must be infinite-dimensional."

In his last published paper [16], that appeared in 2006, Gerard was inspired by Connes' quantization of classical mathematics to develop the essential properties of a still more abstract concept of a Toeplitz operator, for which he constructs a far-reaching index theorem that includes several classical index theorems that pertain to, for example, the Wiener–Hopf integral operator, and almost periodic functions. But he didn't think this was the end of the story, and suspected that one could prove an index theorem in the general setting of what he calls a unimodular algebra on a compact Hausdorff space K, i.e., a function algebra A on K such that every function in $C(K)$ can be approximated by elements of the form $f\theta$, where $f, \theta \in A$ and $|\theta| = 1$.

He also had it in his head in 2005 to draw together his results about Toeplitz operators in a book sometime in the future, but, alas, fate intervened.

6.2. **C^*-algebras.** If for nothing else, Gerard will surely be remembered for his postgraduate-level textbook entitled "C^*-Algebras and Operator Theory", which appeared in 1990. As he writes in the Preface "This book is aimed at the beginning graduate student and the specialist in another area who wishes to know the basics of this subject. The reader is assumed to have a good background in real and complex analysis, point set topology, measure theory, and elementary functional analysis." The book was very well received by the mathematical community worldwide and warmly reviewed, and, according to him, became a standard textbook in many countries. A Russian translation of it appeared in 1997. It's still on sale, and to-date, 1,892 copies of it have been sold.

Not only did its appearance mark Gerard's "arrival" on the international stage, it acted as a springboard for his subsequent professional career, and as a stepping stone to further advancement within and without UCC.

The book deals with the general theory of C^*-algebras, the unifying theme that courses through his work, and was one of his main areas of specialisation. While he studied such structures for their own sake, he was well aware of their origins and importance within Mathematics, their wide range of applications, and the reasons for considering them. Indeed, about a third of his published papers have "C^*-algebras" in their title, and, significantly, about two-thirds of these were published after the appearance of his book. Because these papers are readily identified, a reader wishing to know his major achievements in this area, is recommended to read the book to learn the foundations, and then use MathSciNet to locate his papers, and learn about the recent development of the subject and the directions it has taken as a result of his pioneering investigations.

But to give a flavour of his work which had a major impact in the theory of general C^*-algebras, it seems only right that I should comment on one topic which he studied, and which grew out of his investigations of non-classical Toeplitz operators. In his first paper about these objects [9], his most important idea was the identification of a certain C^*-algebra, $C^*(\Gamma^+)$, where Γ^+ is the positive cone of a discrete ordered abelian group Γ, with a corner of a crossed product of a commutative C^*-algebra by Γ. This led him to generalise the notion of a crossed product of a C^*-algebra A by an abelian group G, which he proceeded to develop in a series if papers, focusing on a theory of semigroup crossed products. His 1996 paper [13] is possibly the most influential of these. In the classical theory and for the simplest situations, one has a given group homomorphism α from G to the group $\mathrm{Aut}(A)$ of *-automorphisms of A. The crossed product should be a C^*-algebra B that contains A and admits a representation τ of G into the unitary elements of B so that $\alpha_g(a) = \tau(g)a\tau(g)^*$. The algebra B should be generated by A and the image $\tau(G)$. Gerard was motivated to his generalisation by more or less contemporary work of P. J. Stacey and of Iain Raeburn with various coauthors. They were in turn partly motivated by a desire to find an underlying theory for the 1977 construction by Cuntz of new simple C^*-algebras, and another motivation for Gerard was in connections with Toeplitz algebras. In [13] he found an appropriate generalisation to the case when the group G was replaced by a cancellative abelian semigroup M with a zero element and the map α is replaced by an action $x \mapsto \alpha_x$ of M as injective *-endomorphisms of the C^*-algebra A. A key step is to find an appropriate larger C^*-algebra where the more classical case of automorphisms reappears and the group G is the Grothendieck group for the semigroup M. This is achieved by an inductive limit construction. And furthermore the crossed product can be twisted by a multiplier of M.

7. Advancement in Cork, 1990–2000

Within a short time after the appearance of his book, Gerard was promoted to Statutory Lecturer in Mathematics at UCC, and shortly after that, in recognition of his scholarly standing, he was honoured by members of the Royal Irish Academy who elected him to membership of this venerable body. He was immensely proud of his membership of the Academy, and, later on, he became joint Editor-in-Chief of its Mathematical Proceedings, helping to change its format and production, which gave it greater visibility and raised its profile as an international journal.

In 1991, he re-commenced the organisation of two-yearly international conferences in UCC on "Operator Theory and Operator Algebras" for which he received funding from a variety of different sources. In the early years he obtained small amounts of money from the Royal Irish Academy and the Irish Mathematical Society, but his principal source of funding in those days was, somewhat surprisingly, the US Air Force. In later years, he received financial support from EOLAS, FORBAIRT and the EU, which over time became the major sponsor.

The conference he organised in 1995 was one of the events held to mark the 150th anniversary of the founding of University College, Cork, and attracted upwards of 100 participants. Coincidentally, in the same year he was promoted to the rank of Associate Professor in Cork in recognition of the quality and quantity of his research output, the calibre of his teaching, and the overall contribution he made to the running of the Department of Mathematics and the well-being of the College.

Around about the same time, he was also invited to join the EU Operator Algebras Network, and, over two four-year periods, succeeded in attracting substantial funding from the EU which provided conference support, and enabled him to offer worthwhile Scholarships to his postgraduate students, and invite several postdoctoral research assistants to come to UCC and work with him. As a result of his establishing a node of this network here, Cork became an internationally recognised centre of excellence, with Gerard as its leading investigator, not only for the promotion of Operator Algebras, but also for the development of Noncommutative Geometry and Quantum Groups, new subjects of great intrinsic importance, both for Mathematics and Physics.

7.1. **Noncommutative Geometry and Quantum Groups.** Together with a succession of postdoctoral research assistants, Tom Hadfield, Johan Kustermans, Deepak Parashar and Lars Tuset, from the late 1990s onwards, Gerard gave seminars in UCC about these fields, and made important contributions to them, but, before attempting to describe these, I invite you to read Gerard's own descriptions of these subjects:

Regarding noncommutative geometry, he says: "Noncommutative geometry was invented by Alain Connes in the 1980s to provide a quantized calculus that extends the usual de Rham calculus and to provide a geometric tool to deal with the so-called singular spaces that arise so frequently in advanced mathematics and quantum physics. A singular space is a space that is poorly behaved from the point of view of classical mathematics in that the usual tools—measure theory, topology, differential geometry, group theory—do not apply. Examples of singular spaces are the spaces of irreducible representations of discrete groups, spaces of orbits of group actions, spaces of leaves of foliations of smooth manifolds, and the phase space of quantum physics. The solution offered by noncommutative geometry is to replace these spaces by associated noncommutative algebras that encode in a better way the problem one wishes to study. For example, instead of studying a space of group orbits of an action of a group G on a compact space X in the pathological case that the natural quotient topology on the space of orbits is the coarse topology (this frequently happens), one studies a corresponding C^*-algebra $C(X) \rtimes G$, the crossed product by G of the algebra $C(X)$ of continuous functions on X. One can then, for example, study the algebraic topology of the space of orbits by studying the K-theory of $C(X) \rtimes G$.

An important feature of Connes' theory is his quantization of the calculus. Given a function f (or more generally, an element in a suitable noncommutative algebra), the differential of f is defined by $df = [F, f] = Ff - fF$, where F is a suitable operator on a Hilbert space. Note that this is well defined even for functions that are not differentiable in the classical sense. The calculi obtained from this construction are associated to objects called Fredholm modules and these in turn, by means of a Chern character construction, give rise to cyclic cocycles (however, not all cyclic cocycles arise in this fashion). The cyclic cocycles in turn form the cycles for an important new cohomology theory, cyclic cohomology, that generalizes in a profound way classical de Rham homology. This theory has already had deep and important applications to classical mathematics and physics."

He said this about quantum groups: "The theory of quantum groups had its origins in attempts to extend Pontryagin duality theory from the context of abelian locally compact groups to non-abelian ones—it turns out that the dual of a non-abelian group is not itself a group; rather, it is a new kind of object called a quantum group. However the class of quantum groups is much more extensive than merely the class of group duals—quantum groups arise in many other ways, for example, by quantizing classical groups to obtain deformations, something that is very important in applications to physics. Quantum groups also arise as symmetry objects for quantum spaces. To explain this latter idea, note that in the framework of noncommutative geometry spaces are replaced by noncommutative algebras that are viewed as quantum spaces. The symmetries of a classical space are analyzed in terms of groups but the symmetries of a quantum space require a quantum group formulation. In the 1980s revolutionary work by S. L. Woronowicz and the Fields Medalist V. G. Drinfeld, arising from considerations in theoretical physics, led to major advances in the theory of quantum groups and to its being regarded as one of the most important subjects in contemporary mathematics. It is envisaged by some physicists that quantum groups will provide the mathematical framework for the solution of the outstanding difficulty of modern physics: the problem of unifying the presently inconsistent theories of general relativity and quantum physics.

In Woronowicz's approach a quantum group is a C^*-algebra with additional structure, such as a comultiplication. Corresponding to a locally compact group the C^*-algebra encodes the topological or geometric aspect and the comultiplication corresponds to the group operation. This theory has been most successfully worked out in the case of compact quantum groups, where Woronowicz has shown the existence of a Haar integral and developed the corepresentation theory. He has also considered the problem of endowing these quantum groups with suitable differential structures. This is an aspect of the theory that is still very mysterious and it is one of the aspects of quantum group theory in which my research is based."

Working jointly with J. Kustermans and L. Tuset, both of whom spent time at UCC, Gerard introduced certain linear functionals called *twisted graded traces*, and developed an extensive theory for them and their integrals. For example, they showed that if one associated a multilinear function φ to a triple $(\Omega, d, \int)$, where (Ω, d) is an N-dimensional differential calculus and $\int$ a suitable twisted graded trace, by setting

$$\varphi(a_0, \ldots, a_N) = \int a_0 da_1 \cdots da_N,$$

then φ is a cycle for a new cohomology theory. They also showed that the new theory had all the basic features of Connes' cyclic cohomology theory and contained it as a special case. Thus, it was shown that a significant portion of Connes' noncommutative geometry can be extended to the case of the differential calculi that arise in the quantum group setting. It should be mentioned, too, that Gerard attached great significance to this new "twisted cyclic cohomology" that they had introduced, and had every intention of following it up before his untimely illness. He seemed to think that the most important thing to be done to further develop the theory was to construct a Chern character.

In a different direction, together with E. Bédos and L. Tuset, he wrote three papers about the concepts of amenability and coamenability in compact quantum groups and algebraic quantum groups, framing the definition of amenability in a way that is analogous to the classical one—a locally compact group G is *amenable* if there is a positive linear functional $m : L^\infty(G) \to \mathbb{C}$ of norm one such that

$$m(\lambda_x f) = m(f) \qquad (f \in L^\infty(G)),$$

where $\lambda_x f(g) = f(x^{-1}g)$, $g \in G$. (Compact groups and abelian groups are amenable.) Their notion of co-amenability stems from a property possessed by a certain C^*-algebra associated with a discrete group. They gave several equivalent formulations of this concept, involving a C^*-algebra. *Inter alia* it turns out that co-amenability of an algebraic quantum group G implies amenability of its dual $\hat{G}$. It's not known if the converse holds.

8. Miscellaneous Research Items

8.1. **Occasional papers.** About 24 of Gerard's published papers have nothing much in common with either Toeplitz operators or C^*-algebras, and about a third of these were joint efforts with others, such as T. T. West (5), M. Mathieu (1), C. K. Fong (2) and K. Abodayeh (1). While it's hard to classify them, they fall into the general area of Spectral Theory of operators on Banach spaces, and show his versatility to work with others on diverse problems. He had the capacity to spot connections between different areas, and oftentimes produce easier proofs of known results. As an illustration, let me single out [14]. According to its reviewer, Qing Lin, "In this elegant short paper, Murphy provides an elementary and much simpler proof" of a theorem of P. Y. Wu to the effect that, in an infinite-dimensional Hilbert space, any unitary operator is a product of 16 positive operators. This is a by-product of the spectral theorem, and a result that, if u is a unitary element in a C^*-algebra, then the matrix $\begin{pmatrix} u & 0 \\ 0 & u^* \end{pmatrix}$ is a product of eight positive ones.

8.2. **Survey Articles.** Gerard clearly enjoyed writing expository papers, and he published at least eight of these, most of them in the Bulletin of the Irish Mathematical Society. These were mainly concerned with surveys of areas that he was currently working in or about to explore, but were aimed at non-specialists. For instance, in issues of the Bulletin he wrote about "Extensions and K-theory of C^*-algebras" (1987), "Toeplitz operators" (1989), "Dimension theory and stable rank" (1990), "C^*-dynamical systems and invariance algebras" (1991), "Partially ordered groups" (1992), and "Function algebras" (1993). An exposition of his work on Quantum Groups is given in [15]. While a common thread runs through these papers, as with most of his work, namely, the abstract theory of C^*-algebras, nevertheless, these are distinctly different, and contain not only historical summaries of separate aspects of the origins and early development of this subject, but serve as useful signposts of further developments. As well, they illustrate his breadth of knowledge and understanding of many areas of mathematics and physics, and provide a valuable insight to his thinking and mode of work.

9. Final Years, 2000–2006

Gerard did his fair share of departmental and College administration during his time in UCC. He served on the College's Promotions Board for many years, and, following the retirement of Paddy Barry from the Chair of Mathematics in 1999, he became Head of Department, a role he filled with quiet efficiency for the next five years, during which time he oversaw the development of new management structures and the delivery of new mathematical degree programmes which attracted bright students from the start, and have grown in popularity, becoming the flagship degree programmes of the School of Mathematical Sciences at UCC. All the while, too, while performing his administrative duties, he continued to teach his courses, produce a steady stream of research papers, maintain the link with the EU Operator Algebras Network, do his editorial work for the Proceedings of the Royal Irish Academy, supervise research students and, in 2003, revive the Cork international conferences on C^*-algebras. Gerard organised his last international conference[5] in June, 2005. A few months later he was diagnosed with cancer.

Gerard was very widely read, and delved deeply into History and Economics, especially. Indeed, he had every intention, apparently, of writing an Economic History of Ireland, and had written copious notes in print form—which was his style of writing—which he hoped to pull together in book form at some stage. Another plan of his was to write children's stories, many of which he composed for his own children, of whom he was exceedingly proud.

[5]Stephen Wills, who co-organised the 2003 and 2005 conferences with Gerard, will run the next one in 2008 with the support of Martin Mathieu and Richard Timoney.

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