ABSTRACTS

Following are the abstracts of contributions (i.e., talks and posters) to the 13th Annual Meeting of the IMS held at the National University of Ireland Maynooth, 6–8 September 2000. The abbreviations after the names designate: 'M' for *main speaker*, 'S' for *speaker*, 'RS' for *research student*, and 'P' for *poster*. The abstracts are included as provided by the contributors for this volume of the *Bulletin*, edited by the editor if deemed necessary.

Harmonic Maps in Unfashionable Geometries

F.E. Burstall, University of Bath (M)

Many special surfaces in classical differential geometry are characterised by the property that an appropriate Gauss map is harmonic and then the integrable features of their geometry (spectral deformation, Bäcklund transformation, algebro-geometric solutions and so on) can be inferred from those of harmonic maps.

Thus a special case of the Ruh–Vilms theorem asserts that a surface has constant mean curvature if and only if its Gauss map is a harmonic map into the 2-sphere and, similarly, a surface has constant negative Gauss curvature if and only if its Gauss map is Lorentz harmonic with respect to the metric induced on the surface by the second fundamental form.

It is interesting that harmonic maps into pseudo-Riemannian symmetric spaces also arise in this context: a surface in S^n is Willmore if and only if its *conformal Gauss map* is harmonic. This is a map into the indefinite Grassmannian that parametrises 2-spheres in S^n and geometrically represents a congruence of 2-spheres having partial second order contact with the Willmore surface.

In this talk I report on work in progress with Udo Hertrich-Jeromin and show that similar constructions are available in Lie sphere and projective differential geometry. Moreover, both geometries can be treated at the same time in a practical manifestation of the celebrated line-sphere correspondence of Lie. In both cases, surfaces in 3-space are studied via their lifts into a contact manifold which can be viewed as the space of lines in a quadric. From this lift we construct a "Gauss map" taking values in a certain Grassmannian which can be viewed as a congruence having partial third order contact with the underlying surface. In Lie sphere geometry our Gauss map is the Lie congruence of cyclides while in projective differential geometry it is the Lie congruence of quadrics.

The Gauss map we construct is conformal in an appropriate sense and its energy integral defines a functional on the underlying surface whose extremals are the minimal surfaces of Lie sphere and projective geometry. I give a simple, conceptual and uniform argument to show that a surface is minimal in Lie sphere or projective geometry precisely when its Gauss map is harmonic.

Decoding a Class of Alternant Codes for the Lee Metric

Eimear Byrne, National University of Ireland Cork (S)

We investigate a class of Lee-metric alternant codes defined over the ring of integers modulo p^n. We give a lower bound on the minimum Lee distance of a code satisfying a certain set of constraints and outline two decoding procedures. The first algorithm operates completely over a Galois ring R, while the second must be implemented over the corresponding residue field. In both cases the decoding schemes are constructed from the perspective of the theory of Gröbner bases in $R[x]^2$ and centre on solving a key equation. We examine the behaviour of certain linear sequences in R in order to characterise M, the module of all solutions to the key equation. We also establish the minimality of the required solution in the solution module, ensuring its presence in a Gröbner basis of M.

Old and New on the Bass Note, the Torsion Function and the Hyperbolic Metric

Tom Carroll, National University of Ireland Cork (M)

The talk consists of a survey of results on the fundamental frequency of a simply connected domain, including Hayman's Theorem, on the torsion function from elasticity theory as well as its occurence in probability theory as the expected lifetime of Brownian motion killed on the boundary of the domain and on the hyperbolic metric, in particular the Bloch constant. Both the simply connected case and the much better understood convex case are discussed. Special emphasis is placed on open problems and in particular on extremal domains for inequalities involving these quantities and the inradius of the domain, that is, the supremum radius of all disks contained in the domain.

Drawing of Stokes Lines for the WKB Solutions of a Second Order Ordinary Differential Equation

Dónal Dowling, Dublin City University (P)

The Stokes phenomenon occurs when multivalued exponential functions are used as WKB approximate solutions to entire functions. The geometry of the Stokes and anti-Stokes lines is important in order to know which values Stokes multipliers will have in different regions of the complex plane. These lines can be drawn by solving a pair of two-dimensional system of first order ordinary differential equations.

Singularly Perturbed Parabolic Problems on Non-Rectangular Domains

Ray Dunne, Dublin City University (P)

A singularly perturbed time-dependent convection-diffusion problem is examined on certain non-rectangular domains. The nature of the boundary and interior layers that arise depends on the geometry of the domains. For problems with different types of layers, various numerical methods are constructed to resolve the layers in the solutions. Various numerical results and graphs are presented that verify experimentally that the the numerical solutions generated by the methods converge uniformly to the solution of the continuous problem with respect to the singular perturbation parameter.

Entropy of Endomorphisms of $P^k(C)$

Julien Duval, Université de Toulouse (M)

We prove, using ideas by Gromov and Lyubich, the uniqueness of the measure entropy for endomorphisms of $P^k(C)$.

Harmonic Maps between Riemannian Polyhedra

Jim Eells, Cambridge University (M)

Maps which are extremals (e.g., minima) of the Dirichlet integral are said to be harmonic. These are ubiquitious in Riemannian geometry. In this lecture my aim is to introduce harmonic maps between certain singular spaces (mainly Riemannian polyhedra)—and to provide geometric and analytic examples, both as domains and as targets—but for simplicity in this lecture, mostly as domains. As examples, 1) branched Riemannian covers, 2) normal complex analytic spaces (embedded in complex projective space), 3) leaf spaces of Riemannian foliations of Riemannian manifolds. Our emphasis is on existence (the direct method of variational theory) and regularity (extensive adaption to harmonic maps of the theory of De Giorgi).

This work is in collaboration with Bent Fuglede, Copenhagen, and will appear as a Cambridge Mathematics tract.

Rank, Rigidity and Symmetry

Jost Eschenburg, University of Augsburg (M)

Symmetric spaces form a subject of central importance in geometry. We report on a class of theorems characterizing Riemannian symmetric spaces of higher rank. These theorems shed light on various aspects of these spaces and bring together very different geometric theories: Riemannian manifolds of nonpositive curvature, spherical Tits buildings and submanifold geometry. In particular, we put emphasis on the third aspect which is related to isotropy orbits of symmetric spaces.

Differential Equation Models for Aujeszky's Disease Virus in the Irish Pig Population

Glenn Fingleton, Dublin City University (P)

Aujeszky's Disease virus, (ADV) is a contagious viral disease that affects the central nervous system of all animals, but swine are its natural host. ADV is a very important economic problem in Ireland, where substantial losses are incurred in the farming community each year. We consider various differential equation models of ADV with homogeneous and proportional mixing between seropositives and seronegatives. We derive expressions for the basic reproduction ratio R_0, and the infectious contact rate. Using these, we perform equilibrium and stability analysis for both non vaccinated and vaccinated models. We find that it may be possible for there to be two endemic equilibria (one stable and one unstable) for $R_0 < 1$. With the possibility of future trade restrictions, brought about by EU regulations, a nationwide eradication programme could be beginning shortly. Implications for control/eradication strategies are considered.

Aspects of the Embeddability Ordering in Topology

Michael Gormley, Queen's University Belfast (RS)

The embeddability ordering is one of the most natural notions in topology and is defined as follows: Let α be an infinite cardinal number (the case where α is finite is trivial). Let $T(\alpha) = \{\mathcal{T}_i : i \in I\}$ be the set of (homeomorphism classes of) topologies definable on α. We say that $(\alpha, \mathcal{T}_i)$ embeds into $(\alpha, \mathcal{T}_j)$ if $(\alpha, \mathcal{T}_i)$ is homeomorphic to a subspace of $(\alpha, \mathcal{T}_j)$. We define a quasi-order on $T(\alpha)$ as follows: $\mathcal{T}_i \leq \mathcal{T}_j$ if $(\alpha, \mathcal{T}_i) \hookrightarrow (\alpha, \mathcal{T}_j)$, that is, $(\alpha, \mathcal{T}_i)$ embeds into $(\alpha, \mathcal{T}_j)$. Given $\mathcal{T} \in A \subseteq T(\alpha)$ we say that $\mathcal{T}$ is weakly quasi-minimal in A if $(\mathcal{T}' \in A, \mathcal{T}' \leq \mathcal{T}) \Rightarrow \mathcal{T} \leq \mathcal{T}'$. We say that $\mathcal{T}$ is strongly quasi-minimal (sqmin) in A if $(\mathcal{T}' \in A, \mathcal{T}' \leq \mathcal{T}) \Rightarrow \mathcal{T}' = \mathcal{T}$.

Example: Let $(\alpha, \mathcal{I}(x))$ denote the topology on α whose (non-empty) open subsets are precisely those which contain x. Then we have that $(\alpha, \mathcal{I}(x))$ is sqmin connected.

In $T(\alpha)$ we can construct an antichain of cardinality $2^{2^{\alpha}}$. Hence there does not exist a quasi-maximum member of $T(\alpha)$ (that is a space which contains a copy of every member of $T(\alpha)$). It follows that there does not exist a weakly quasi-maximal member of $T(\alpha)$. It can also be shown that no pair of incomparable topologies has a supremum in $T(\alpha)$. We then ask whether or not $T(\alpha)$ forms an upper semi-multilattice which is defined as follows: Let E be a qoset. Then E is said to form an upper semi-multilattice if, for every $x, y, z \in E$ with $x \leq z$ and $y \leq z$, there exists a minimal upper bound w to x and y with $w \leq z$. We can show that $(T(\alpha), \leq)$ does not form an upper semi-multilattice (nor a lower semi-multilattice). This involves defining a topology on the union of a family of non-empty pairwise disjoint topological spaces indexed by a partially ordered set.

Extreme 2-Homogeneous Polynomials on Real ℓ_p Spaces

Bogdan Grecu,
National University of Ireland Galway (RS)

In the space of 2-homogeneous polynomials on a real Hilbert space we consider the problem of determining the extreme points of the unit ball. In this setting the problem can be solved completely. Since every Hilbert space is an ℓ_2 space, we attempt to solve the problem for other ℓ_p spaces. It turns out that the case $p > 2$ provides some interesting examples. We analyse in detail the case of the 2-homogeneous polynomials on the two-dimensional ℓ_p spaces and describe completely its unit ball emphasising the huge difference between this and the result on a two-dimensional Hilbert space.

On the Expected Density of Complex Roots of Random Polynomials

Alexander V. Grigorash, University of Ulster (P)

The poster describes a method to study the asymptotics of the expected density of complex roots of random polynomials.

Random polynomials considered have complex coefficients. The real and imaginary parts of each coefficient are normal distributed (Gaussian) random variables. All the random variables under consideration are independent.

In the most general case the method presented allows to obtain the mathematical expectation of the density of complex roots of equation $F_n(z) = K$, where $F_n(z)$ is a random polynomial of degree n, and K is a given complex number. The derivation of the expected density of roots is shown for random algebraic polynomials. Other random polynomials can be transformed to algebraic class. The transformation is shown for random hyperbolic and trigonometric polynomials.

Of special interest is the behaviour of the expected density of roots when the polynomial degree n is high. The finite formulae for the limiting value of the expected density when $n \to \infty$ are shown for two classes of random polynomials: generalised hyperbolic polynomials with standard normal coefficients, and algebraic polynomials with identically distributed normal coefficients. For the latter class of polynomials, the behaviour of this limiting value is illustrated graphically with three-dimensional representations.

The Coating Profile on a Finite Substrate After Dip-Coating

Michael Hayes, University of Limerick (RS)

Certain products in industry need to be coated with a uniform thin layer of coating, e.g. fluorescent tubes need to be coated with a thin layer of phosphorus. The layer has to be uniform so as the tubes' light emission will not change along the length of the tube. This process is mathematically modelled to find the thickness of the coating and to investigate if various parameters were changed would that improve the uniformity of the coating thickness, e.g. reduce edge effects.

Dip-coating is a process which involves submersing the substrate that needs to be coated in a suspension of the coating material (e.g. phophorus) and an inert liquid (e.g. water). The substrate is then removed from the suspension. During evaporation of the solvent, the liquid runs down due to gravity, influencing the final coating profile.

The method of characteristics is used to find solutions for the fluid thickness and solid profile after evaporation of the solvent. An explicit solution for the liquid is found analytically. In the case of the solute profile, two initial value problems are solved—an infinite and finite initial fluid thickness.

The characteristics for the solute profile in the case of the problem with an infinite initial fluid thickness are obtained analytically by solving a non-linear ordinary differential equation. A renormalisation technique is implemented to find a finite initial condition. After scaling the expression for the solute profile is simplified.

A different set of characteristics which are piece-wise are found in the case of the finite initial fluid thickness problem. Self similar solutions are found after scaling.

The solutions for the solute profile are presented and discussed.

A Unified Approach to Covariant and Lie Differentiation

Donal Hurley, National University of Ireland Cork (S)

A differentiation operator with respect to a smooth vector field X on a smooth manifold $\mathcal{M}$, such as covariant differentiation ∇_X or Lie differentiation $\mathcal{L}_X$, satisfies the following basic requirements; its action is $\mathbf{R}$-linear on tensor fields, it preserves tensor rank, it commutes with contractions of tensors, it satisfies the Leibniz rule for differentiation of tensor products, and the action on smooth functions is given by directional differentiation. It then follows that such a type of differentiation is determined by its action on smooth vector fields.

If $\{\mathbf{e}_{(i)}\}$ and $\{\mathbf{e}^{(i)}\}$ denote dual basis for the tangent bundle $T\mathcal{M}$ and the cotangent bundle $T^*\mathcal{M}$, then using the Einstein summation convention, the covariant derivative $\nabla_X Y$ of a vector field $Y = Y^i \mathbf{e}_{(i)}$ in the direction of $X = X^i \mathbf{e}_{(i)}$, can be written as $\nabla_X\, Y = \{X(Y^i) + {}_\nabla\Lambda^i{}_j(X)\}\,\mathbf{e}_{(i)}$ where ${}_\nabla\Lambda^i{}_j(\mathbf{e}_{(k)}) = \Gamma^i{}_{jk}$ are the connection coefficients and ${}_\nabla\Lambda^i{}_j(fX) = f\,{}_\nabla\Lambda^i{}_j(X)$ for any smooth function f on $\mathcal{M}$. The Lie derivative $\mathcal{L}_X\, Y$ is given by $\mathcal{L}_X\, Y = \{X(Y^i) - {}_\mathcal{L}\Lambda^i{}_j(X)\,Y^j\}\,\mathbf{e}_{(i)}$ where ${}_\mathcal{L}\Lambda^i{}_j(\mathbf{e}_{(k)}) = -\,\mathbf{e}^{(i)}\,[\mathbf{e}_{(j)}, \mathbf{e}_{(k)}] = -\,D^i{}_{jk}$, the commutation coefficients, and ${}_\mathcal{L}\Lambda^i{}_j(fX) = f\,{}_\mathcal{L}\Lambda^i{}_j(X) - (\mathbf{e}_{(j)} \otimes \mathbf{e}^{(i)})\,(df, X)$.

Generalizing these, we define D-differentiation to be the family of differentiations with respect to X where given by

$$D_X Y = \{X(Y^i) + \Lambda^i{}_j(X)\; Y^j\}\,\mathbf{e}_{(i)}$$

with $\Lambda^i{}_j(fX) = f\Lambda^i{}_j(X) - A^i{}_j{}^a{}_b\mathbf{e}_{(a)}\,(f)X^b = f\Lambda^i{}_j(X) - A^i{}_j\,(df, X)$ for some family of $(1,1)$ tensors $A^i{}_j$.

A vector field V is said to be *euthygrammic* if $D_V V = 0$, and the integral curve of such vector fields are called *euthygrammes*.

One application of this general differentiation is given by an investigation of the trajectories of electrons in a crystaline lattice which is not subject to external forces. The trajectories are euthygrammes where the corresponding D-differentiation is, in general, neither covariant nor Lie differentiation.

Details of this joint work with M. Vandyck, Physics Department, UCC, can be found in J. Phys. A **33** (2000), 6981.

Computing Invariant Manifolds of Maps

Patrick Johnson, University of Limerick (P)

The project is concerned with theoretical and numerical methods of computing invariant manifolds of discrete dynamical systems which are described by 2-D maps. The theoretical side seeks to extend Hadamard's analytical approach to the calculation of invariant manifolds of 2-D maps with particular reference to Garcia's Map. The numerical simulation of this and other maps are investigated using the techniques of Hobson, You Kostelich and Yorke and Chua and Parker. Non-invertible maps are being investigated with attention being paid to their Critical Curves in an attempt to better visualise and understand their behaviour. Some particular maps are also being investigated using existing software packages to investigate and compare the accuracy of these packages in computing and graphing the dynamics of the maps.

The Elliptic Curve Method for Factoring

Seamus Kelly,
National University of Ireland Maynooth (RS)

In 1985 Hendrik Lenstra introduced the Elliptic Curve Method for Factoring. The algorithm was an analogue of Pollard's $p-1$ Method. Using the ECM allows one to use many elliptic curves of different orders. Pollard's Method is much less flexible however as it restricts the user to one group order; and this gives the elliptic curve method a considerable advantage when searching for factors. Since 1995, many changes have taken place to speed up the original ECM. For example, one can work in projective space to avoid inversions. Also it is possible to define different addition laws on the elliptic curve that take less time to run than the standard addition laws that one may have come across in text books such as Bressoud and Koblitz. Finally, to increase the probability that the group order is smooth (which we would like!), one can use a particular type of curve whose group order is divisible by certain small primes.

Elementary Operators on Calkin Algebras

Martin Mathieu, Queen's University Belfast (S)

Let A be a (unital, complex) Banach algebra. For every pair of n-tuples $a = (a_1, \dots, a_n)$, $b = (b_1, \dots, b_n) \in A^n$, the corresponding elementary operator on A is defined by

$$S\colon x \mapsto \sum_{j=1}^{n} a_j x b_j \qquad (x \in A).$$

Properties of elementary operators, especially on algebras of operators, have been vigorously studied over the past decades. It emerges that the situation of the Calkin algebra (that is, the quotient of all bounded operators on a Banach space E by the ideal of all compact operators on E) is full of surprises. In this setting, an elementary operator with dense range is surjective, and injectivity entails boundedness below. In the case of Hilbert space, positivity implies complete positivity, and the norm and the *cb*-norm of every elementary operator coincide. We present an overview on some results of this flavour, in particular on recent extensions of the latter two results to antiliminal-by-abelian C^*-algebras (obtained by Archbold, Somerset, and the speaker), and strong rigidity properties of the norm of elementary operators on Calkin algebras due to Saksman and Tylli.

Some Properties of the Neumann Operator

Violeta Morari,

National University of Ireland Cork (RS)

Let $g\colon \mathbb{R}^n \to \mathbb{R}$, $n \geq 2$ be a Lebesgue measurable function. By the

Neumann Kernel N_y, we mean

$$N_y(x) = \frac{1}{(\|x\|^2 + y^2)^{\frac{n-1}{2}}}.$$

For $y > 0$, we define G as the convolution between g and the Neumann Kernel for the half space $\mathbb{R}^{n+1}_+$, i.e.,

$$(g \star N_y)(x) = G(x, y) = \int_{\mathbb{R}^n} g(u) N_y(x - u) du.$$

Some of the mapping properties of the operator $M_y : g \to G$ as well as some analytic properties of the function G are discussed. These include:

- G is harmonic in the half space $\mathbb{R}^{n+1}_+$ for $g \in L^p$, $1 \leq p < n$.
- $M_y : L^r \to L^t$ is bounded for $1 \leq r < n$ and $t > \frac{nr}{n-r}$.
- For $r = 1$, the norm can be determined exactly.
- For $p = \frac{n}{n-1}$, there exists at least one family of integrable functions S_ξ for which the norm of the convolution converges to infinity as $\xi \to 0$.
- Any function g harmonic in the half space $\mathbb{R}^{n+1}_+$ and bounded and continuous on its closure can be represented as a convolution between g and $\frac{\partial}{\partial y} N_y$.

Lexicodes and the Chain Condition

Katie O' Brien,
National University of Ireland Cork (RS)

Error correcting codes are used to ensure that digital information transfer is able to withstand noise and other forms of corruption. A lexicographic code or lexicode is constructed by applying a greedy algorithm to a lexicographically ordered vector space. We consider lexicodes and their relationship with the chain condition. A code C is said to satisfy the chain condition if there exists a chain of subcodes $D_1 \subseteq D_2 \subseteq \cdots \subseteq D_k = C$ such that the dimension of D_i, $\dim(D_i) = i$ for $1 \leq i \leq k$ and each D_i has minimum effective length of all i-dimensional subcodes of C.

The chain condition property has developed from and in parallel with other important coding theory topics such as generalised hamming weights, dimension length profiles, and trellises. Using an alternative construction for the lexicode we have information about the covering radius. We use this to analyse the properties of lexicodes pertaining to the chain condition. Some necessary conditions for a code to satisfy the chain condition are examined and we show that lexicodes satisfy these. Also we give a direct proof that lexicodes with minimum distance 3 and 4 satisfy the chain condition. Further, we show that Forney's claim (IEEE Trans. IT-40, 1994) that lexicodes necessarily satisfy the chain condition is incorrect.

Pervasive Algebras of Analytic Functions

Alejandro Sanabria,
National University of Ireland Maynooth (P)

Let X be a compact Hausdorff topological space and $C(X,\mathbb{C})$ (respectively, $C(X,\mathbb{R})$) the Banach algebra of all continuous complex-valued (respectively, real-valued) functions on X endowed with the uniform norm. A *function space* S on X is a closed subspace of $C(X,\mathbb{C})$. Denote by $\operatorname{clos}_{C(E,\mathbb{C})}S$ (respectively, $\operatorname{clos}_{C(E,\mathbb{R})}S$) the closure in $C(E,\mathbb{C})$ (respectively, $C(E,\mathbb{R})$) of the function space (respectively, *real subspace*) S, where E is a closed subset of X.

Let Y be a closed subset of X. A function space S on X is said to be *complex pervasive* on Y if $\operatorname{clos}_{C(E,\mathbb{C})}S = C(E,\mathbb{C})$ whenever E is a proper non-empty closed subset of Y. Similarly, a real subspace S of $C(X,\mathbb{R})$ is said to be *real pervasive* on Y if $\operatorname{clos}_{C(E,\mathbb{R})}S = C(E,\mathbb{R})$ whenever E is a proper non-empty closed subset of Y.

Let U be an open subset of the Riemann sphere $\hat{\mathbb{C}}$ and denote by $\operatorname{bdy} U$ its topological boundary. We consider the case when $X = \hat{\mathbb{C}}$, $Y = \operatorname{bdy} U$ and S coincides with the algebra $A(U)$ of all complex-valued functions continuous on $\hat{\mathbb{C}}$ and analytic on U, or with Re $A(U)$, the space of real parts of elements of $A(U)$. We give a complete characterization of complex pervasiveness in topological terms. This is not possible for real pervasiveness. We give a complete characterization involving continuous analytic capacity.

Analogue results to the complex pervasive case are still valid on open Riemann surfaces when the open set U has compact closure.

[1] I. Netuka, A.G. O'Farrell and A. Sanabria-García, *Pervasive algebras of analytic functions*, J. Approx. Theory, to appear.

[2] A.G. O'Farrell and A. Sanabria-García, *Pervasive algebras of analytic functions on Riemann surfaces*, Proc. Roy. Irish Acad., to appear.

First Steps Towards a Theory of Spectrally Bounded Operators

Gerhard Schick, Queen's University Belfast (RS)

The concept of a spectrally bounded operator was introduced by M. Mathieu in 1994. A linear mapping T from a subspace E of a Banach algebra A into another Banach algebra is called *spectrally bounded* if there exists a constant $M \geq 0$ such that $r(Tx) \leq Mr(x)$ for all $x \in E$, where $r(\cdot)$ denotes the spectral radius. Resembling strongly the notion of a bounded linear operator between Banach spaces, this concept calls for a thorough investigation, which is the topic of my PhD thesis. In the present talk, I report on first steps in this direction.

In the first part, I sketch the historical development starting with Frobenius' studies of spectrum-preserving mappings on matrix algebras in 1897. Around 1970, Kaplansky raised the question whether every surjective spectrum-preserving linear mapping onto a semisimple Banach algebra has to be a Jordan homomorphism. The most recent (positive) answer to this question is due to Aupetit for the case of von Neumann algebras. Analogous questions for spectrally bounded operators are presently under consideration [1].

In the second part, I discuss some basic properties of spectrally bounded operators and the relation to bounded operators. A fundamental result of Aupetit entails the boundedness in case the spectrally bounded operator maps onto a semisimple Banach algebra. In order to try to develop a duality theory, spectrally bounded linear functionals are collected into a 'spectral dual', endowed with the 'spectral operator norm'. However, in general non-commutative algebras, there is no analogue of the Hahn-Banach theorem. This and other difficulties are mainly due to the bad behaviour of the spectral radius.

[1] M. Mathieu and G. Schick, *Spectrally bounded operators from von Neumann algebras*, preprint 2000.

Surrogate Forms of Central Simple Algebras

THOMAS UNGER,
NATIONAL UNIVERSITY OF IRELAND DUBLIN (RS)

Given a central simple algebra A over a field F, one can associate to it the so-called trace form T_A. Properties of the quadratic form T_A reflect properties of A. If the characteristic of F is 2, this is no longer true since T_A has rank zero. We introduce a new quadratic form S_A, the surrogate form of A, and show that if $\mathrm{char}(F) \neq 2$ the forms S_A and T_A give the same amount of information [2]. When $\mathrm{char}(F) = 2$, S_A is nondegenerate and has similar properties as when $\mathrm{char}(F) \neq 2$ [1]. In "most" cases, S_A is a good replacement for T_A in any characteristic.

[1] G. Berhuy and C. Frings, *On the second trace form of central simple algebras in characteristic two*, preprint 2000.

[2] T. Unger, *A note on surrogate forms of central simple algebras*, Math. Proc. Royal Ir. Acad., to appear.

Geometry of Q-reflexive Banach Spaces

MILENA VENKOVA,
NATIONAL UNIVERSITY OF IRELAND DUBLIN (RS)

Let E be a Banach space, and let $P({}^nE)$ be the space of all continuous n-homogeneous polynomials on E endowed with the topology of uniform convergence on the unit ball B of E. The Aron-Berner extension operator $AB_n\colon P({}^nE) \longrightarrow P({}^nE'')$ extends every n-homogeneous operator on E to an n-homogeneous polynomial on E''.

Consider the mapping $J_n\colon \widehat{\bigotimes}_{n,s,\pi} E'' \longrightarrow P({}^nE)'$, where $[J_n(\otimes_n x)](P) = [AB_n(P)](x)$ for all $P \in P({}^nE)$ and all $x \in E''$. The space E is called *Q-reflexive* if J_n is an isomorphism for all n. If E is Q-reflexive then the spaces $P({}^nE)''$ and $P({}^nE'')$ are isomorphic.

Theorem. *Let E be a Q-reflexive Banach space and suppose E'' has the approximation property. Then*

(1) *c_0 (and hence ℓ_∞) is not a subspace of $\widehat{\bigotimes}_{n,s,\epsilon} E' = (\widehat{\bigotimes}_{n,s,\pi} E)'$ for any n;*

(2) *E' does not contain a copy of ℓ_p for any $1 \le p \le \infty$;*

(3) *E is not B-convex;*
(4) *ℓ_1 is finitely represented in E.*

The Geometry of Unfoldings

C.T.C. Wall, University of Liverpool (M)

Let $G\colon N \to P$ be a holomorphic map between complex manifolds; write

$$\Sigma(G) := \{x \in N \mid dG_x \text{ is not surjective}\}$$

for the singular set of G and $\Delta(G) := G(\Sigma(G))$ for its discriminant. Write θ_N for the module of vector fields on the source N, θ_P for the module of vector fields on the target P, and $\theta(G)$ for the module of vector fields along G. Here we think of germs of vector fields at a point.

We have a diagram of tangent bundles

$$\begin{array}{ccc} TN & \xrightarrow{TG} & TP \\ \big\downarrow & & \big\downarrow \\ N & \xrightarrow{G} & P \end{array}$$

and interpret θ_N as the set of sections to the projection $TN \to N$, θ_P as the set of sections to $TP \to P$, and $\theta(G)$ as the set of maps from N to TP making the lower triangle commute. Composition (on the right) with TG defines a map $tG\colon \theta_N \to \theta(G)$; composition (on the left) with G defines a map $\omega G\colon \theta_P \to \theta(G)$. We say that ξ *lifts* η if $tG(\xi) = \omega G(\eta)$.

The geometrical properties of the discriminant are simplest in the case when G is a stable map: roughly, G is stable if, at any $\mathbf{x} \in N$, $tG(\theta_N) + \omega G(\theta_P) = \theta(G)$; and the restriction of G to $\Sigma(G)$ is a finite, proper map. There is a convenient 'normal form' for stable maps, where we start with a fairly arbitrary map f_0 and use explicit rules to write down a 'versal unfolding'.

One may regard a stable map $G\colon N \to P$ as the collection of its restrictions $G|G^{-1}(y)$ to the fibres. We partition P according to the types of singularity on the corresponding fibres, and call the parts *leaves*.

The following major results (due to many authors) hold for stable maps with $\dim N > \dim P$, and indeed much more generally.

(i) The discriminant $\Delta(G)$ is a reduced complex space of dimension $p-1$ and $G|\colon \Sigma(G) \to \Delta(G)$ is its normalisation.
(ii) A vector field on P is liftable under G if and only if it is tangent to the discriminant $\Delta(G)$.
(iii) The leaf L_y containing $\mathbf{y}$ is smooth there, with tangent space given by the values at $\mathbf{y}$ of the liftable vector fields.
(iv) The codimension of L_y equals the sum τ_y of the Tjurina numbers of the singularities of G at the points of $\Sigma(G) \cap G^{-1}(\mathbf{y})$.
(v) The space of vector fields tangent to $\Delta(G)$ is a free module over the ring of functions on P.
(vi) If $G_1\colon N_1 \to P$ and $G_2\colon N_2 \to P$ are stable, with $\Delta(G_1) = \Delta(G_2)$, then there is a holomorphic equivalence $\phi\colon N_1 \to N_2$ with $G_2 \circ \phi = G_1$.

[1] E.J.N. Looijenga, *Isolated singular points on complete intersections*, London Math. Soc. Lecture Notes no. **77**, Cambridge University Press, 1984.
[2] A.A. du Plessis and C.T.C. Wall, *Discriminants and vector fields*, Singularities: the Brieskorn anniversary volume (V.I. Arnold, G.-M. Greuel and J.H.M. Steenbrink, eds.), Birkhäuser, 1998, pp. 119–140.

The Argument Principle and Boundaries of Analytic Varieties

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We consider a simple closed curve γ in $\mathbb{C}^n$, and we assume that γ is smooth and oriented.

When does there exist a Riemann surface Σ in $\mathbb{C}^n$ having γ as its boundary?

A necessary condition is given by the Maximum Principle for functions analytic on Σ. Let P be a polynomial on $\mathbb{C}^n$ and let x be a point of Σ. The restriction of P to Σ is analytic, so $|P(x)| \le \max_\gamma |P|$. It follows that, if Σ exists, the polynomial hull $\hat{\gamma}$ of γ is distinct from γ. So we have the necessary condition: $\hat{\gamma} \neq \gamma$.

In the 1960's the following result was proved:

Curve Theorem. *If $\hat{\gamma} \neq \gamma$, then $V = \hat{\gamma} \backslash \gamma$ is an analytic variety V.*

Later it was shown that the boundary of V (in the sense of Stokes' Theorem) equals γ.

Let now M be a smooth compact oriented manifold in $\mathbb{C}^n$ with dimension $2q-1$.

Problem. When does there exist an analytic variety V, $\dim_{\mathbb{C}} V = q$, with boundary of V equal to M?

R. Harvey & B. Lawson gave necessary and sufficient conditions in 1975, in terms of the structure of the tangent spaces T_x to M.

Recently, in the paper "*Linking Number and Boundaries of Analytic Varieties*", Annals of Math. **151** (2000), pp. 125-150, Herbert Alexander and the author returned to this problem. Later on, we modified our result and gave a condition on M which uses the non-negativity of the degree of certain polynomial maps. The following Theorem is proved in a forthcoming paper entitled "The argument principle and boundaries of analytic varieties", which is to appear in the memorial volume for Bela-Szokefalvi Nagy, which is about to be published.

Theorem. *Let M be a smooth, compact, oriented manifold in $\mathbb{C}^n$ of dimension $2q-1$. Assume the following: For every polynomial map $P = (P_1, \ldots, P_q)$ with $P \neq 0$ on M and $\dim_{\mathbb{C}}\{P = 0\} = n - q$, the degree of the map: $f = \frac{P}{|P|}$, of M to the sphere $\mathbb{S}^{2q-1}$ in $\mathbb{C}^q$, is non-negative. Then there exists an analytic subvariety V of $\mathbb{C}^n \setminus M$ such that $M = \operatorname{bd} V$, in the sense of Stokes' Theorem.*