PROBLEM PAGE

Editor: Phil Rippon

Ray Ryan has had the excellent idea of numbering the problems according to the issue in which they appear. This should make it much easier to keep track of earlier problems. For example, the problems in the previous Bulletin will be referred to from now on as Problems 19.1, 19.2 and 19.3.

This relabelling exercise reminds me of the story about the Real Analysis textbook which contained an apology by the author for numbering the sections before he had defined the positive integers!

I came across the first problem this time at an Open University Summer School. A student there was tormenting the Maths tutors with it (and also with Problem 18.1, whose solution appears below).

20.1 *Find a formula, whose value is 64, which uses the integer 4 twice and no operations other than:*

$$+, -, \times, /, \uparrow, \sqrt{\ } \text{ and } !$$

I gather that Míchéál Ó Searcóid has devised a formula for 64 which uses only one 4, but this requires the functions [ ] and ln as well. Also, it is possible to display 64 on a calculator (more precisely, on some calculators) using a single 4 followed by the four key strokes: ×× ==. This was pointed out by the daughter (aged 14) of one of my colleagues here at the OU.

Next, here is another problem from my colleague John Mason.

20.2 *Given n positive integers $a_k, k = 1, 2, \ldots, n$ (not necessarily distinct), prove that some sum of the form*

$$a_{k_1} + a_{k_2} + \ldots + a_{k_m}, \qquad 1 \le k_1 < k_2 < \ldots < k_m \le n,$$

is equal to 0 mod n.

Finally, a pretty geometric problem which I heard from Harold Shapiro some years ago.

20.3 *Show that if a square lies within a triangle, then its area is at most half the area of the triangle.*

Now for solutions to the problems in Issue 18.

18.1 *Find the next entry in the following sequence:*

$$1, 11, 21, 1211, 111221, 312211, \ldots.$$

This problem has circulated widely in recent years and provoked much interest and anguish! To understand the pattern, think of any given term, such as 1211, as a string of positive integers and then describe this string in the form:

one one, one two, two ones.

This description yields the next string: 111221. The string following 312211 is, therefore, 13112221.

As mentioned in Issue 18, this problem is associated with John Conway who has made a remarkable investigation of the behaviour of sequences determined by this process (which he calls "Audioactive Decay"). This is written up in Eureka, vol 46, 1986, the journal of the Archimedeans, which is the Cambridge students mathematical society.

A string of digits is said to *split* if it can be written as a product LR of strings L and R, such that

$$(LR)_n = L_n R_n, \qquad \text{for } n = 1, 2, \ldots.$$

Here L_n denotes the nth descendant of L under this process. A string with no non-trivial splitting is called an *element*. Conway lists a sequence of 92 *common elements* (each with an appropriate name) all of which are involved in the descendants of every string, except 22 (hydrogen) and the empty string. Furthermore, eventually only the common elements (and possibly a few 'isotopes') are involved.

For example, the 7th descendant of the string 1 is

11132.13211,

which is written here as the product of the two common elements Hafnium and Stannium. All descendants of this compound involve only the 92 common elements.

Finally, apart from the two exceptions above, the lengths of strings involving common elements increase exponentially at a universal rate $\lambda = 1.3035\ldots$ and the relative abundances of the common elements in such strings tend to fixed positive values.

18.2 ***Find an infinite family of pairs of distinct integers m, n such that m,n have the same prime factors, and $m-1, n-1$ have the same prime factors.***

This problem came from the book 'It seems I'm a Jew' by Freiman and Nathanson, which chronicles the methods which have apparently been used to discriminate against Jewish students in the Soviet Union. By setting high school students fiendishly difficult problems in oral exams, it seems that even the most able can be excluded for the prestigious Faculty of Mechanics and Mathematics at Moscow State University. In an appendix to the book, Andrei Sakharov discusses such problems, including the one above, which he describes as 'incredibly difficult' at this level. The solution

$$m = 2^k - 1, \quad n = \left(2^k - 1\right)^2, \qquad k = 1, 2, \ldots,$$

is easy to verify, once you've seen it.

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