department inspectors. There was a third level representative on the Senior Cycle Committee.

There are questions still to be answered: What course should those first year pupils follow who started in September 1987 and who would normally have done the Group Certificate in 1990? Will calculators be allowed in time? And will the next Junior Cycle review be even more democratic than the last one? But we can at least raise our hats to the first syllabus in Irish schools which will have a certificate at three levels

References

[1] D.J. Hurley and M. Stynes, *Basic Mathematical Skills of UCC Students*, IMS Bulletin 17 (1986), 68–75.

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The Theory of Blunders!

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We all come across mathematical blunders of all types and sizes when correcting scripts, answering questions, during discussions or when checking homework. Very often, these blunders can be corrected with no recurrence by convincing the students of the error of their ways e.g. a frequent error which occurs in different guises is $\frac{1}{a} + \frac{1}{b} = \frac{1}{a+b}$, so ask them to work out $\frac{1}{3} + \frac{1}{5}$ and $\frac{1}{3+5}$.

What happens on many occasions is that the student fails to stop and think that perhaps something he or she has been doing all his or her mathematical life, and getting away with it, may be incorrect, and in fact *utterly* false. A student at one time came up to me having failed the exam totally convinced he should have passed. I looked up his script and discovered that everywhere he should have integrated he differentiated and everywhere he should have differentiated he integrated, and nearly all done correctly! He flew through the exam at the next attempt. Why hadn't I spotted this during the year? (I have a reason, closely approaching an excuse!)

We don't expect such blunders from a student in our small honours classes, but they still occur and we can be on the lookout by marking some work before the official examination. We haven't anywhere approaching the resources to sort out these problems in our large pass classes. Unfortunately very often the first time we see some of our students' work is at the end of the year and then it is too late. What we need to do is take in work regularly, go through it ourselves and return the work *individually* pointing out errors and asking that problems, similar to those where the errors occurred, be attempted and handed in again for checking. Of course this is impossible with the very large numbers we have to cater for e.g. this year I have some classes of approximately 180, 130 and 100 students and to give this kind of attention to even one of these would take up all of my time, with no lectures anywhere else.

Is there a solution? One solution, not necessarily unique, would be to double the staff numbers in our Mathematics Departments, but of course this is impractical without even considering our present economic climate. (I have presented a solution so as a Mathematician need I go any further?) From my own experience of teaching small classes here and abroad I am totally convinced that with the proper tuition most of the glaring blunders can be eliminated.

What I think we need to do as a first step is to convince the students that such blunders occur, in fact they *themselves* do make such errors and, to really get it to sink in, that such blunders will lose lots of marks thereby dramatically increasing the probability of failure. Explain the difference between a blunder and a slip, which all of us make from time to time. This hopefully will produce a type of self-reconsideration and discussion amongst themselves and with us and tutors when available. Whenever I point out a silly error in class there is always a great commotion, thinking perhaps it is a joke on some poor individual, but the class is never convinced when I point out that over 40% of them made that particular error on last year's exam.

I thought I'd try something out on this year's first year pass class. In order to set the background, the first year pass class at U.C.G., excluding Engineers, is broken into two groups, a "fast" stream getting 3 hours of lectures a week and a "slow" stream getting 4 hours of lectures a week, but both leading to the same examination. The streams are divided very roughly by the Leaving Certificate or Matriculation results, those with apparently weaker results going into the "slow" stream. I had the "slow" stream so I tried the true-false test reproduced below on these apparently weaker students but I feel the results would not have deviated very much had all the first year class been included. This true-false test most consisted of blunders I frequently encountered with a few other items thrown in just for discussion or fun. (Note in particular that question 19 is one of the fun ones which certainly produced some reaction and discussion.)

The test was given at the beginning of the lecture and students were informed that they had 40–45 minutes in which to complete it. They were asked to keep a record of their answers on the question sheet for discussion later and answer sheets were to be completed anonymously. All had finished within 35 minutes and answer slips were collected. For the rest of that class and for all of the next, I went through the quiz demonstrating where possible why such a statement was false (by e.g. assuming the statement was true and then proving that $0 = 1$). There were interesting discussions during and especially after each class.

Following the discussions with the students and reading through the questions now, I can see a number of improvements that could be made, but in the interests of accurate statistics, they are reproduced here as given. I am sure others have examples of their favourite and frequently occurring blunders and

I would be most happy to hear about these. I would be interested also to hear of other methods, tried or untried, on how to try to eliminate these glaring errors which occur right up to degree level.

Previous articles [1, 2, 3] report on deficiencies in the mathematical skills of our students but the reader will appreciate the differences between what is discussed in these and what is contained here although of course the two are interconnected in many ways.

The Questions

Which of the following are true(T) and which are false(F)?

1. $\frac{1}{a}+\frac{1}{b}=\frac{1}{c} \Rightarrow a+b=c$.
2. $x>y \Rightarrow \frac{1}{x}>\frac{1}{y}$, for all $x, y, x \neq 0, y \neq 0$.
3. $0^0 = 1$.
4. $x>y \Rightarrow \frac{1}{x}<\frac{1}{y}$ for all $x, y, x \neq 0, y \neq 0$.
5. $(x^2+y^2)^2 = x^4+y^4$.
6. $\sin ax = a \sin x$.
7. $\infty + \infty = \infty$ and $\infty - \infty = 0$.
8. $1^0 = 1$.
9. $\frac{1}{2} \div \frac{1}{2} = \frac{1}{4}$.
10. A function always has an inverse.
11. If $9+h^2=64$ then $h = \pm\frac{8}{3}$.
12. $x>0 \Rightarrow \frac{1}{x}<0$.
13. $\frac{0}{0}=1$.

14. $\dfrac{(x-1)(x+1)}{(x-1)(x+2)} = \dfrac{2}{3}$ when $x = 1$.

15. $3.(x+y)^{-1} = 3.x^{-1} + 3.y^{-1}$.

16. $-(x^2+4x+4) = -x^2+4x+4$.

17. $a^{-2} = \dfrac{1}{\sqrt{a}}$.

18. $(a^2)^3 = a^5$.

19. This question is false!

20. $\log x + \log x^2 = 3 \log x$.

21. $+\sqrt{0.04} = .02$.

22. The solution set of the equation $x(x+2) = 0$ is $x = -2$.

23. $(\sqrt{x})^{1/3} = (x^3)^{1/2}$.

24. $\dfrac{3(x-2)}{x^2-4} = \dfrac{3}{4}$ when $x = 2$.

25. $60^\circ = \dfrac{\pi}{3}$ radians.

26. If $x^2 < 4$ then $x < \pm 2$.

Percentage Responses

Question	% answering True	% answering False	% not answering
1:	32%	68%	0%
2:	8%	92%	0%
3:	44%	55%	1%
4:	86%	13%	0%
5:	13%	89%	0%
6:	32%	65%	3%
7:	68%	31%	1%
8:	62%	38%	0%
9:	20%	80%	0%
10:	61%	36%	3%
11:	13%	87%	0%
12:	34%	66%	0%
13:	17%	83%	0%
14:	31%	69%	0%
15:	57%	42%	0%
16:	2%	98%	0%
17:	58%	40%	2%
18:	63%	37%	0%
19:	61%	22%	17%
20:	48%	49%	3%
21:	25%	73%	3%
22:	86%	15%	0%
23:	29%	67%	4%
24:	1%	98%	1%
25:	84%	13%	3%
26:	76%	23%	1%

There is a story (true or false?) that a certain teacher, nameless of course, in a certain school, nameless also, informed the school inspector that he/she advised his/her Intermediate Certificate class to *always* choose B in the multiple choice part of the Mathematics paper as he/she had done a survey of the previous few years and B had come up more often than any other. I believe there will be no multiple choice questions when the new Intermediate Certificate syllabus is examined.

References

[1] N. O'Murchu and C.T. O'Sullivan, *Mathematical Horses for Elementary Physics courses*, I.M.S. Newsletter, 6(1982), 50-54.

[2] *Report on the Basic Mathematical skills test of First Year Students in Cork RTC in 1984*, I.M.S. Newsletter, 14(1985), 33-43.

[3] Donal Hurley and Martin Stynes, *Basic Mathematical skills of U.C.C. students*, Bull. I.M.S. 17(1986), 68-75.

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NOTES

Wedderburn's Theorem Revisited (Again)

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In a previous note in this Bulletin [3] we proved the following theorem which generalises the theorem of Wedderburn that a finite division ring is a field.

Theorem 1 *Let R be a ring with unity. If more than $|R| - \sqrt{|R|}$ elements of R are invertible, then R is a field.*

The bound $|R| - \sqrt{|R|}$ is the best possible because of the existence of $\mathbf{Z}_{p^2}$, which has exactly $p^2 - p$ invertible elements for any prime p, but yet is not a field.

Another formulation of Wedderburn's theorem is the following: If R is a finite ring with unity and every non-zero element of R is invertible, then R is commutative.

This naturally leads to the following question: If R is a finite ring with unity, can we force the conclusion that R is commutative by assuming that a proper subset of the non-zero elements are invertible? The purpose of this note is to prove the following:

Theorem 2 *Let R be a finite ring with unity. If every non-zero ring commutator $[x, y] = xy - yx$ of R is invertible then R is commutative.*

Proof Let $c = [x, y] \neq 0$. Consider the sequence $c, c^2, c^3, \ldots$ Since R is finite, $c^i = c^j$ for some $j > i \geq 1$. By hypothesis, c is invertible, so $c^{j-i} = 1$ and thus $c^{j-i+1} = c$. R now satisfies the hypothesis of a theorem of Herstein [1], $[a, b]^{n(a,b)} = [a, b]$ for $n(a, b) \geq 1$. If we are prepared to invoke the full power of this theorem, it follows at once that R is commutative. Alternatively, we can use the following more elementary result of Herstein [2]: If R is a finite ring in which every nilpotent element is cenmtral, then R is commutative.